Time Lotteries and Stochastic Impatience*

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Abstract

We study preferences over lotteries in which both the prize and the payment date are uncertain. In particular, a *time lottery* is one in which the prize is fixed but the date is random. With Expected Discounted Utility, individuals must be risk *seeking* over time lotteries (RSTL). In an incentivized experiment, however, we find that almost all subjects violate this property. Our main contributions are theoretical. We first show that within a very broad class of models, which includes many forms of non-Expected Utility and time discounting, it is impossible to accommodate even a single violation of RSTL without also violating a property we termed Stochastic Impatience, a risky counterpart of standard Impatience. We then present two positive results. If one wishes to maintain Stochastic Impatience, violations of RSTL can be accommodated by keeping Independence within periods while relaxing it across periods. If, instead, one is willing to forego Stochastic Impatience, violations of RSTL can be accommodated with a simple generalization of Expected Discounted Utility, obtained by imposing only the behavioral postulates of Discounted Utility and Expected Utility.

Key words: Time Lotteries, Stochastic Impatience, Risk and Time preferences, Expected Discounted Utility.

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1 Introduction

Consider the choice between *(i)* receiving a prize in period $\bar{t}$ for sure, or *(ii)* receiving the same prize in a random period t with mean $\bar{t}$. For example, the choice may involve receiving a desirable outcome (such as \$100 or a dinner at a fancy restaurant) in 10 weeks for sure versus either 5 or 15 weeks with equal probability. Both options deliver the same prize and have the same expected delivery date, but in one of them the date is uncertain. What would, or should, one choose? More generally, what are individuals' attitudes towards uncertainty not only about *which* outcome will be realized, but also about *when* this outcome will be received? Are they captured by existing models and, if not, how can they be modeled? How do they interact with other notions of risk and time preferences? These are the questions that this paper studies.

Many economic decisions involve uncertainty about both which and when outcomes will be received. Home sellers are typically uncertain not only about the sale price but also about how quickly the house will be sold. When starting a new project, investors do not know how much dividends the project will pay and when they will be paid. Looking for a job involves uncertainty about which job will be found, and when. Both dimensions of uncertainty, and possibly their interaction, matter for choice in these and other domains. In many cases the uncertainty pertains only, or predominantly, to timing. For example, one may know she will inherit a specific house, but not when. Similarly, with public housing and other government-provided services, much of the uncertainty concerns the time these will be provided. When shopping online, is it worthwhile to pay an additional fee to ensure delivery at a guaranteed rather than random date? All such choices depend on one's attitudes towards randomness in time.

We introduce a notion of risk attitudes towards lotteries in which the timing is random: time lotteries. Just like regular (atemporal) risk aversion is defined by a preference for a sure amount over a lottery with the same expected value fixing the date, the individual is Risk Averse Over Time Lotteries (RATL) if she prefers to receive a fixed prize in a sure date rather than in a random date with equal expected delay. She is Risk Seeking over Time Lotteries (RSTL) if she displays the opposite pattern. While these definitions compare payments in a sure vs. a random date, they have implications for when both lotteries are random, the mean dates are not equal, or both prizes and dates are random. To capture attitudes toward time lotteries based on intrinsic preferences and not on planning motives, we assume that all uncertainty is resolved as soon as one of the options is chosen.[1]

Our starting point is the observation that the standard model in economics, Expected Discounted Utility (EDU), prescribes that all subjects must be globally RSTL. To see this, note that if $u(x) > 0$ is the utility of prize x and $\beta \in (0, 1)$ is the dis-

[1] If instead uncertainty resolved gradually over time, the absence of risk could improve planning. We want to separate this instrumental benefit from intrinsic preferences.

count factor, then the value of receiving x at time t is $\beta^t u(x)$, whereas the value of the lottery with a random date is $\mathbb{E}[\beta^t]u(x)$. Since β^t is convex in t, the latter option is always preferred. Note that the curvature of the utility function over prizes plays no role in this comparison. This aspect of the standard model has important implications, with many papers relying crucially and explicitly on it (e.g., Ely and Szydlowski 2017; Zhong 2019).[2]

As the standard model makes such sharp predictions, we start our investigation by testing them in incentivized experiments. (Previous work, discussed below, only used unincentivized surveys.) These predictions fail to hold in our data: only a small fraction is consistently RSTL; instead, most subjects are risk *averse* in the majority of questions. Attitudes towards time lotteries are also correlated with standard risk aversion, an intuitive connection that is missing from EDU.

In light of these findings, we present a series of theoretical results on modeling general attitudes towards uncertainty in both prize and date. We confine our analysis to preferences over lotteries of dated rewards, that is, pairs of the form (x, t), where x is a monetary prize and t is the time in which it is received.[3]

Our first theoretical question is: Can we have a model that allows for *at least one violation* of RSTL, while keeping other desirable properties?[4] To this end, we consider a new property, called Stochastic Impatience. Intuitively, it requires that, when facing lotteries that pay different outcomes in different periods, the individual prefers to associate higher payments to earlier dates. Consider two prizes, say \$100 and \$20, and two periods, say a day and a month. Stochastic Impatience requires the individual to prefer the 50/50 lottery that pays either \$100 in a day or \$20 in a month, over the 50/50 lottery that pays either \$100 in a month or \$20 in a day. As such, it is a risky counterpart to the standard notion of Impatience; under EDU, Impatience and Stochastic Impatience imply each other. Note that the choice above can be rephrased as: Given a base lottery paying \$20 either in a day or a month, which prize do we want to increase by \$80, the earlier or the later one? To the extent that receiving earlier payments is better, the individual should prefer the first option, in accordance with Stochastic Impatience.

The answer to our question is an impossibility result about the coexistence of violations of RSTL and Stochastic Impatience. The result holds under very mild assumptions: when there is no risk, individuals like receiving higher payments, dislike waiting, and are not future biased (their preferences may be stationary or present bi-

[2]For example, Zhong (2019) studies dynamic models of information acquisition and shows that the optimal strategy uses Poisson signals. The reason is that the decision time with Poisson signals is a mean-preserving spread of other strategies and is thus preferred under EDU due to RSTL.

[3]We focus on this setup for simplicity. As we show in Appendix A, extensions to lotteries over streams are immediate. It is also easy to generalize our results to an arbitrary set of outcomes.

[4]We are not suggesting that individuals should be globally RATL. Indeed, they may change their attitude when some of the prizes occur in a distant future. For example, subjects may prefer the lottery between today and two years, over one year for sure due to present bias. They may also prefer a lottery between one year and one thousand years over five-hundred years for sure, as only the closest date is conceivable. In intermediate cases, however, we may see instances of RATL.

ased, but not the converse); and when evaluating risky prospects, they follow a Local Bilinear model (a very general form that includes Expected Utility and common non-Expected Utility models, such as Cumulative Prospect Theory or Disappointment Aversion). With these general assumptions, we show that Stochastic Impatience implies RSTL. That is, under very general conditions satisfied by virtually all models used in practice, there is a fundamental incompatibility between Stochastic Impatience and even a single violation of RSTL.

Given this impossibility, our second question is: Is there a plausible way to generalize EDU to allow for at least some violations of RSTL? We give two solutions, the merit of each depends on the importance one attaches to the properties being relaxed.

First, we show that violations of RSTL together with Stochastic Impatience can be accommodated by relaxing Independence in a way specific to the intertemporal setup: maintaining it within but not across periods (so that it also violates Bilinearity). To illustrate this, we provide an example of a functional form—borrowed from the finance literature—with these features. As we discuss in Section 4.1, this approach is related to the order in which time and risk are aggregated.

Second, we show how RSTL can be accommodated while maintaining full Independence if one is willing to forego Stochastic Impatience. In fact, this can be achieved by keeping all the main properties used to motivate EDU. While EDU can be seen as merging the *functional form* of Discounted Utility without risk with the one of Expected Utility, we instead impose the properties that characterize each of them. A simple result is that these properties together do not characterize EDU, but lead to a more general model, that we call Generalized EDU (GEDU), in which there is a strictly increasing utility function over prizes u, a discount factor β, and a strictly increasing function ϕ, such that preferences are represented by

$$\mathbb{E}\big[\phi\big(\beta^t u(x)\big)\big].$$

GEDU is similar to models in the literature (in particular, Kihlstrom and Mirman 1974 applied to time). Unlike EDU, GEDU can accommodate different attitudes towards time lotteries, with the individual being RATL if and only if ϕ is more concave than the log function. More generally, and in accordance with our experimental findings, a more concave ϕ implies that subjects are both more risk averse in an atemporal setting and more averse to time lotteries.

Related Literature. A small literature has discussed attitudes towards time lotteries. Chesson and Viscusi (2003) show that EDU implies a preference for random timing. They hypothesize that risk aversion over time may be due to high risk aversion or hyperbolic discounting. The latter is proven impossible by Onay and Öncüler (2007), who generalize their theoretical results, pointing out that (what we call) RSTL holds for any convex discount function and link it to probability distortions. Chen (2013) also shows that EDU implies RSTL. Ebert (2017) extends the analysis to

higher-order risk preferences (prudence and temperance). We show that RATL can be accommodated within the framework of Expected Utility; at the same time, if one wants to preserve Stochastic Impatience, then not even allowing for probability distortions is sufficient.

For experimental evidence, Chesson and Viscusi (2003) conduct a hypothetical survey with business owners and find that about a third of them are RATL. Onay and Öncüler (2007) run a non-incentivized survey with large hypothetical payments and find that most subjects are RATL. By contrast, Kacelnik and Bateson (1996) show evidence that animals in foraging decisions tend to be RSTL. Eliaz and Ortoleva (2016) show that subjects are ambiguity averse when timing is ambiguous.

2 Risk Aversion over Time Lotteries

Consider an interval of monetary prizes $[w, b] \subset \mathbb{R}_{++}$ and a set of dates $T \subset \mathbb{R}_{+}$ consisting of either non-negative consecutive integers ("discrete time") or an interval of non-negative numbers ("continuous time"), with $0 \in T$ in both cases. The ordered pair $(x, t) \in [w, b] \times T$ denotes receiving an amount of money x in t periods.[5] Let Δ be the set of simple lotteries over $[w, b] \times T$ endowed with the topology of weak convergence, and $\delta_{(x,t)}$ denote the degenerate lottery that gives (x, t) with certainty. We study a complete and transitive preference relation $\succsim$ over Δ, where $\sim$ and $\succ$ denote its symmetric and asymmetric parts, respectively. To avoid trivial cases, when T is discrete, we assume that it has at least three elements and that the outcome space is rich enough so that $\delta_{(b,t+1)} \succ \delta_{(w,t-1)}$ for any t that is neither the minimal nor the maximal element of T. Intuitively, this condition rules out the possibility that either the space of prizes is so small or discounting so strong that getting even the worst prize one period sooner is better than waiting one more period for the best prize.

Both in the theoretical part and in our experiment, we focus on lotteries in which uncertainty is resolved immediately. Therefore, preferences in our setting are static and do not stem from planning considerations. We do so to focus on the starkest case in which only time and risk preferences are at play. While introducing benefits from planning or preferences over the timing of resolution of uncertainty may strengthen the appeal of risk-free options, our goal is to investigate—empirically and theoretically—individuals' intrinsic attitudes towards such uncertainty.

Standard risk aversion is defined by positing that an individual prefers a sure amount to a lottery of the same expected value—all within a fixed date. We take an analogous approach to define risk attitudes towards uncertainty about the time, by considering lotteries that pay a fixed prize x at a random time: we call them *time lotteries*. For example, a time lottery could pay \$100 in either one or two months.

[5] While inconsequential to our mathematical analysis, we interpret outcomes (x, t) as representing goods that are consumed on date t rather then prizes received in a certain date that need not coincide with the time of actual consumption.

Formally, for any $x \in [w, b]$, we say that $p_x \in \Delta$ is a *time lottery with prize* x if $y = x$ for any (y, t) in its support.

Definition 1. The relation $\succsim$ is Risk Averse over Time Lotteries (RATL) if for all $x \in [w, b]$ and all time lotteries p_x with prize x, if $\bar{t} = \sum_\tau p_x(x, \tau) \times \tau$ then

$$\delta_{(x,\bar{t})} \succsim p_x.$$

Analogously, $\succsim$ is Risk Seeking over Time Lotteries (RSTL) or Risk Neutral over Time Lotteries (RNTL) if the above holds with $\precsim$ or $\sim$, respectively.

In words, the individual is RATL if she prefers to receive a certain amount in a sure time to receiving the same amount on a random time with the same mean. RSTL and RNTL are defined analogously. While RATL (and its counterparts) are defined only for comparisons between sure dates and lotteries with the same expected date, in most models they have implications on general attitudes about risk in timing—just like standard risk aversion (defined as a comparison between a sure prize and random one) has implications on general risk preferences.[6]

The standard model to study risk and time is Expected Discounted Utility (EDU), according to which lotteries are evaluated by

$$V(p) = \mathbb{E}_p\big[\beta^t u(x)\big], \tag{1}$$

where u is a positive-valued utility function over money and $\beta \in (0, 1)$ is a discount factor.[7] EDU evaluates a time lottery with prize x as $\mathbb{E}_p\big[\beta^t\big]u(x)$. Since β^t is a convex function of t, it follows from Jensen's inequality that, independently of u, any EDU preference relation must be RSTL.

In fact, this argument holds more generally since it only relies on the convexity of the discount function. Suppose that preferences are represented by

$$V(p) = \mathbb{E}_p\big[D(t)u(x)\big], \tag{2}$$

where D is a strictly positive and strictly decreasing function with $D(0) = 1$. Then, preferences are RSTL if and only if D is convex. All discount functions used in practice—including exponential, hyperbolic, and quasi-hyperbolic—are convex.[8] More-

[6]For example, one may define attitudes towards time lotteries in terms of preferences for mean-preserving spreads in time. Under EDU, and more generally in the GEDU model described in Section 4.2, RATL coincides with an aversion to mean-preserving spreads because of Independence, so these definitions are equivalent.

[7]The utility u must be positive in our setting of lotteries over dated rewards. In settings that include streams, if $u(0)$ is the value of getting nothing, direct calculations show that $u(x)$ should be replaced with $u(x) - u(0) > 0$, leading to the same observation.

[8]See Supplementary Appendix A for the definition of convexity for discrete T. We also test convexity in our experiment and find that the vast majority of subjects satisfy it. Convex discounting is implied by No Future Bias, a property that we discuss in detail in Section 3.

over, when T is unbounded, no strictly decreasing function $D : T \to (0, 1]$ can be concave. Thus, in this case no preference relation represented by (2) can be RATL.[9]

The impossibility of EDU to accommodate different attitudes towards time lotteries can be understood with an analogy to the work of Yaari (1987). Within the (atemporal) Expected Utility framework, diminishing marginal utility of income and risk aversion are bound together via the curvature of the utility function over prizes. But, as Yaari argues, these two properties are "horses of different colors" and hence, as a fundamental principle, a theory that keeps them separate may be desirable. In our setting, convex discounting, which is a property of deterministic settings, implies RSTL, a property of stochastic settings. There is no fundamental reason why the two should be related. Moreover, in Yaari's analysis even though diminishing marginal utility of income and risk aversion relate to two different phenomena, they are both reasonable and documented properties. In our case, however, while convex discounting is a plausible and documented behavioral property, we now provide evidence that most people violate RSTL.

2.1 Experimental evidence of RATL

We now describe the results of incentivized experiments that measure attitudes towards time lotteries. We start with our main experiment, which uses monetary payments, and then briefly discuss a robustness experiment that uses real-effort tasks instead. Because the purpose of this section is primarily to motivate our theoretical results, we postpone in-depth analyses to Appendix D.

Design. 196 subjects took part in an experiment run at the Wharton Behavioral Lab. The experiment has three parts. Part I asks subjects to choose between different time lotteries: two options that paid the same prize at different dates, where the distribution of payment dates of one option was a mean preserving spread of that of the other. For example, the first question asked them to choose between *i*) \$15 in 2 weeks or *ii*) \$15 in 1 week with probability $\frac{3}{4}$ and in 5 weeks with probability $\frac{1}{4}$. Subjects answered five questions of this kind; in two of them, both options involved random payment dates. Table 2 in Appendix D lists the questions.

Parts II and III use the Multiple Price List (MPL) method to measure time and risk preferences separately. Part II measures standard time preferences as well as attitudes towards time lotteries. Part III measures atemporal risk preferences, with payments taking place immediately at the end of the session. These include measures of regular risk aversion, as well as Allais' common-ratio-type questions.

At the end of the experiment, one question was randomly selected for payment. The order of parts and questions was partly randomized, except that all subjects

[9]The risk attitudes towards time lotteries in Definition 1 are defined for arbitrary periods and prizes. In Supplementary Appendix A, we introduce their local counterparts, relate it to the local convexity/concavity of the discount function, and show that preferences represented by (2) must be locally-RSTL in all but a finite number of periods.

Table 1: Attitude Towards Time Lotteries in Part I

% of RATL choices	Long		Short	
Question 1	65.7		56.0	
All questions	60.6		47.9	
Both dates random	69.0		45.6	
# of RATL choices	Long		Short	
	Percent	Cum.	Percent	Cum.
0	2.9	2.9	9.9	9.9
1	9.5	12.4	16.5	26.4
2	22.9	35.2	24.3	50.6
3	23.8	59.0	26.4	76.9
4	28.6	87.6	19.8	96.7
5	12.4	100.0	3.3	100.0

received Part I first, and all subjects received the same first question on a separate sheet of paper. The answer to this question is a key indication of the subjects' preferences, as it captures their reaction uncontaminated by other questions.

We ran two treatments: a long delay treatment ('Long'; 105 subjects) and a short delay treatment ('Short'; 91 subjects). Two questions were identical; in the others, the treatment differed in the length of the delays: the maximum delay was 12 weeks in the Long treatment and 5 weeks in the Short treatment.

Results, RATL. Our main results pertain to the attitude towards time lotteries, elicited in the five questions of Part I. Table 1 presents the percentage of RATL choices in *i*) Question 1 of Part I, *ii*) all questions, and *iii*) only questions in which both option involved random dates. It also shows the distribution of the number of RATL answers that each subject gave (where 0 means never RATL, 5 means always RATL).

In both treatments, only a minuscule (2.86% or 9.89%) fraction of subjects are consistently RSTL; instead, the majority of subjects choose according to RATL in the majority of questions. This pattern holds also in the first question and when both options are risky. As discussed above, these findings are not compatible with EDU.

Significance and Difference Between Treatments. While EDU predicts that all choices should be RSTL, one may consider stochastic extensions that account for randomness in individuals' choice (Luce, 1958; McFadden, 2005). In these models, subjects may prefer the RSTL option, yet occasionally choose the other one 'by mistake.'[10] Is our data compatible with such random extension of EDU? We can

[10]For example, the individual may assign a higher utility to the RSTL option, but choose it with a frequency based on the ratio between the utilities of the two (Luce, 1958). When close to indifference,

reject it, in two ways.

First, in any such stochastic extension, the individual should choose the option with the higher utility more frequently (this is true in any of these models, as long as the error is symmetric); under EDU, this should be the RSTL option. Indeed, even assuming a high level of noise and small utility difference, the RATL option should be chosen *at most* 50% of the time. This is not what we observe: in the first question, for example, the proportion of RATL choices is significantly larger than 50% (Exact Binomial Probability Test, p=0.002).[11]

Second, we can compare behavior across treatments, as some questions were different.[12] In particular, in Q5 of the Short treatment, the prize was $15 and the options were (*i*) 50% in 2 weeks, 50% in 5 weeks vs. (*ii*) 75% in 3 weeks, 25% in 5 weeks. In the Long treatment, option (*i*) was identical, but (*ii*) was replaced by (*ii'*) 75% in 1 week, 25% in 11 weeks. Both (*ii*) and (*ii'*) are mean preserving spreads of (*i*), and should thus be chosen under EDU; but (*ii'*) is also a mean-preserving spread of (*ii*). Under EDU, the utility difference is thus smaller between (*i*) and (*ii*), as in the Short treatment, and larger between (*i*) and (*ii'*), as in the Long treatment. RSTL choices for this question should then be more frequent in the Long treatment. But we find the opposite: RSTL is significantly less frequent in the Long treatment (26.67% vs. 47.25%; Fishers Exact Test, $p = 0.003$). Our data is therefore not compatible not only with EDU, but also with typical random choice extensions of it.

RATL, Convexity, Risk Aversion. Next, we analyze the relationship between RATL, convexity of discounting (measured using the questions on time preferences), atemporal risk aversion, and violations of Expected Utility (see Appendix D.3 for details and statistical analysis). In line with previous findings, 82% of our subjects exhibit convex discounting. We also find that 39.89% choose approximately according to Expected Utility over atemporal lotteries. Focusing on subjects in either of these two groups, RATL is still prevalent, with almost identical proportions as in the entire sample. Regression analysis confirms that certainty bias or convexity of discounting are generally uncorrelated with RATL.[13]

Lastly, we test the relation between the tendency to exhibit RATL and atemporal risk aversion. Here we find a significant correlation: subjects who are more risk averse over money also tend to be more RATL (Table 10 in Appendix D.3). This is intuitive,

the frequency is close to 50/50. This is the case in typical stochastic choice models (Luce, 1958; McFadden, 2005). A recent literature expressed concerns in using these models for risk and time preference (Apesteguia and Ballester, 2018). If, instead, we use the random parameter model that Apesteguia et al. (2017) proposed to deal with these concerns, RSTL should be always chosen.

[11]Moreover, the distribution of answers across the five questions is significantly different from a Binomial distribution (Chi-Square Goodness of Fit, $p = 0.0083$ for Short, $p = 0.0000$ for Long).

[12]For the two questions that were identical (Q1 and Q3), differences in behavior are not significant (Fisher exact test, $p = 0.187$ for Q1; $p = 0.148$ for Q3).

[13]As shown above, this is not compatible with EDU, where RSTL is connected with the convexity of discounting. These results also suggest that RATL may not be due to violations of Expected Utility, as opposed to the hypothesis of Chesson and Viscusi (2003) and Onay and Öncüler (2007).

as both are forms of risk aversion; as we have seen, however, it is hard to reconcile this connection within EDU.

Robustness: Real-Effort Experiment. The experiment described above uses monetary payments. To test the robustness of our main findings, we conducted a separate experiment with real effort (Augenblick et al., 2015), in which time lotteries were over the date of effort relief. The experiment involved button-pressing tasks in three sessions over three weeks with subjects recruited on Amazon Mechanical Turk, as in DellaVigna and Pope (2018). Subjects were asked three incentivized questions involving time lotteries. For example, they could choose between skipping work in week 2 for sure or participating in a lottery that allowed them to skip work in weeks 1 or 3 with equal probability. Here as well we find that a majority of subjects choose the RATL option in the majority of questions, showing the robustness of our findings. Appendix D.4 describes the details of this experiment.

3 Stochastic Impatience and Impossibility Results

We have seen that EDU cannot accommodate even a single violation of RSTL. In this section, we show that under mild assumptions, any violation of RSTL is incompatible with a property that we call Stochastic Impatience.

Stochastic Impatience. Consider the choice between:

A Receive either \$100 today or \$20 in a month, with probability $\frac{1}{2}$ each;

B Receive either \$20 today or \$100 in a month, with probability $\frac{1}{2}$ each.

Both options involve the same prizes, probabilities, and dates, but in the first one the higher prize is associated with the earlier date, keeping the same odds. One could imagine that, to the extent that the individual prefers higher payments sooner, this option should be preferred. An appealing argument for this property could be made by decomposing each alternative into two parts. Observe that both A and B offer the same basic lottery in which the individual receives \$20 either today or in a month. The difference between them is which payment is increased by \$80: option A increases today's, while option B the payment in a month. Insofar as earlier is better, option A should be preferred. Indeed, this property can be seen as a counterpart of Impatience for risky environments.

Formally, *Stochastic Impatience* requires that if (in the presence of risk) the individual can choose to pair each monetary payment to a different delivery time, she would pair the highest outcome with the earliest date. A companion paper, Dillenberger et al. (2018), discusses its relation with models that separate risk and time

preferences.[14]

Definition 2 (Stochastic Impatience). The relation $\succsim$ satisfies Stochastic Impatience if for any $t_1, t_2 \in T$ and $x_1, x_2 \in X$ with $t_1 < t_2$ and $x_1 > x_2$,

$$\frac{1}{2}\delta_{(x_1,t_1)} + \frac{1}{2}\delta_{(x_2,t_2)} \succsim \frac{1}{2}\delta_{(x_1,t_2)} + \frac{1}{2}\delta_{(x_2,t_1)}.$$

Basic Properties. We now introduce basic properties we want preferences to satisfy. To keep our analysis as general as possible, we consider the following three conditions over degenerate lotteries (i.e., when there is no risk):

Axiom 1 (Outcome Monotonicity). *For all $x, y \in [w, b]$ and $s \in T$, if $x > y$ then $\delta_{(x,s)} \succ \delta_{(y,s)}$.*

Axiom 2 (Impatience). *For all $x \in [w, b]$ and $s, t \in T$, if $t < s$ then $\delta_{(x,t)} \succ \delta_{(x,s)}$.*

Axiom 3 (No Future Bias). *For all $x, y \in [w, b]$, $s, t \in T$ with $t < s$, and $\tau > 0$ with $s + \tau, t + \tau \in T$, if $\delta_{(x,t)} \sim \delta_{(y,s)}$ then $\delta_{(x,t+\tau)} \precsim \delta_{(y,s+\tau)}$.*

The first two axioms are standard monotonicity properties, requiring that the individual likes higher payments and dislikes delays. No Future Bias states that the individual's willingness to pay to bring consumption earlier by a fixed time length does not increase in time. That is, the rate of substitution between money and time is non-increasing in time. This property generalizes standard notions of Stationarity, that requires the rate of substitution to be constant in both money and time, thus allowing present bias but ruling out the opposite (future bias). No Future Bias is a weak requirement, widely documented empirically and satisfied by most models used to study time preferences (not only exponential discounting, but also hyperbolic or quasi-hyperbolic, developed to account for present bias).[15]

Together, these three conditions impose very few restrictions on preferences. For example, they do *not* require preferences over sure outcomes to be represented by a multiplicatively separable utility function $U(x, t) = D(t)u(x)$. This flexibility even allows for properties such as the magnitude effect (Noor, 2011), where preferences over sure outcomes are represented by $U(x, t) = (\beta(x))^t u(x)$, for some increasing function $\beta(x)$.

Expected Utility. We first present our results assuming that subjects follow Expected Utility, imposing Independence and a continuity assumption.

[14] Our definition below focuses on lotteries with only two outcomes in their support. Our results remain unchanged if the definition is strengthened to lotteries with n equally likely outcomes. We focus on this version for simplicity and because we want the weakest condition for our results.

[15] Even though the most empirical work finds no future bias and this is typically assumed in theoretical studies, there are empirical studies that document future bias in specific contexts (Loewenstein, 1987; Attema et al., 2010; Takeuchi, 2011; Andreoni and Sprenger, 2012).

Axiom 4 (Independence). *For all $p, q, r \in \Delta$ and $\lambda \in (0,1)$,*

$$p \succsim q \;\Leftrightarrow \lambda p + (1-\lambda) r \succsim \lambda q + (1-\lambda) r.$$

Axiom 5 (Continuity). *For all $p \in \Delta$, the sets $\{q \in \Delta : p \succsim q\}$ and $\{q \in \Delta : q \succsim p\}$ are closed.*

It is easy to see that under Axioms 1-5 preferences are represented by

$$V(p) = \mathbb{E}_p\big[U(x,t)\big], \tag{3}$$

where U is continuous, strictly increasing in x, strictly decreasing in t, and such that for $t < s$ and $\tau > 0$, $U(x,t) = U(y,s)$ implies $U(x, t+\tau) \leq U(y, s+\tau)$.

Note that under Expected Utility, Stochastic Impatience can be rewritten as

$$\frac{1}{2}U(x_1,t_1) + \frac{1}{2}U(x_2,t_2) \geq \frac{1}{2}U(x_1,t_2) + \frac{1}{2}U(x_2,t_1),$$

that is, U satisfies decreasing differences:

$$U(x_1,t_1) - U(x_1,t_2) \geq U(x_2,t_1) - U(x_2,t_2). \tag{4}$$

Thus, Stochastic Impatience states that the utility lost by postponing a prize for one period is higher for higher prizes, and thus the utility is maximized by pairing higher prizes with earlier dates and lower prizes with later ones. Put differently, Stochastic Impatience states that prizes and waiting times are substitutes in U. It is then easy to see why Stochastic Impatience must be satisfied by any model, including EDU, where $U(x,t) = D(t)u(x)$ with decreasing D. Indeed, in that case, Stochastic Impatience is equivalent to Impatience (decreasing D).

Impossibility Under Expected Utility. We are now ready to state our first impossibility result.

Theorem 1. *Suppose $\succsim$ satisfies Axioms 1-5. Then Stochastic Impatience implies RSTL.*

The result above shows that, under the conditions of Axioms 1-5, it is impossible to have even a single violation of RSTL without violating Stochastic Impatience. For an intuition, suppose time is continuous and preferences are represented as in (3) with a twice differentiable U. No Future Bias means that the marginal rate of substitution between money and time is weakly decreasing, i.e.,

$$\frac{\partial}{\partial t}\left[-\frac{\frac{\partial U}{\partial t}}{\frac{\partial U}{\partial x}}\right] \leq 0.$$

Stochastic Impatience means that money and time are substitutes, so the marginal disutility of waiting is increasing in the prize ($\frac{\partial^2 U}{\partial x \partial t} \leq 0$). But if the denominator is

decreasing (Stochastic Impatience) and the ratio of marginal utilities is increasing (No Future Bias), then the numerator must be increasing in t: we must have $\frac{\partial^2 U}{\partial t^2} \geq 0$. This means that the marginal disutility of waiting is decreasing, which is precisely RSTL.

Another way to understand Theorem 1 is through the lenses of risk aversion. Recall Options A and B in the discussion of Stochastic Impatience, and observe that the two prizes offered by Option B (20 today or 100 in a month) are, in terms of desirability, strictly in between the two prizes offered by Option A (100 today or 20 in a month). In utility terms, Option A has a higher mean but is 'more spread out.' With EDU, only expected utility matters, not its spread, so Option A is strictly preferred. But an individual who is averse to such spread in utils may prefer to 'hedge' between the two, choosing Option B instead. By requiring that the individual prefers Option A, Stochastic Impatience posits that she cannot be too averse to spreads in utils. But it is precisely this aversion that allows for instances of RATL: the theorem shows that to accommodate even one of them, the aversion to spreads must be such that Stochastic Impatience is violated.

We conclude with a sketch of the proof. The proof of Theorem 1, just like the one of Theorem 2 below, is constructive: if we observe an instance of RATL, we can design a choice problem with similar prizes and delivery times, in which the individual violates Stochastic Impatience. To illustrate, suppose $\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)}$ for some $x \in [w, b]$, $t_1, t_2, t_3 \in T$ with $t_2 = \frac{t_1+t_3}{2}$. For the purpose of this sketch, suppose that we can find $x' < x$ such that $\delta_{(x',t_1)} \sim \delta_{(x,t_2)}$. By No Future Bias $\delta_{(x',t_2)} \precsim \delta_{(x,t_3)}$. By Independence and by the assumption above, $\frac{1}{2}\delta_{(x',t_1)} + \frac{1}{2}\delta_{(x,t_2)} \sim \delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)} \succsim \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x',t_2)}$. Thus, $\frac{1}{2}\delta_{(x',t_1)} + \frac{1}{2}\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x',t_2)}$, violating Stochastic Impatience. The complete argument appears in Appendix C, which collects all proofs of results in the main text.

Beyond Expected Utility. Having seen that any violation of RSTL is incompatible with Stochastic Impatience within Expected Utility, it is natural to ask whether such impossibility holds outside this class. We now show that this is indeed the case.

Consider a model that we call *Generalized Local Bilinear Utility* (GLBU). This model replaces Expected Utility with the much weaker assumption of Local Bilinearity: it posits that 50/50 lotteries between (x, t) and (x', t'), where $\delta_{(x,t)} \succsim \delta_{(x',t')}$, are evaluated by weighting the utility of (x, t) by $\pi(\frac{1}{2})$ and that of (x', t') by $(1 - \pi(\frac{1}{2}))$, and summing them up. When $\pi(\frac{1}{2}) = \frac{1}{2}$, the model coincides with Expected Utility (for 50/50 lotteries); if, instead, $\pi(\frac{1}{2}) < \frac{1}{2}$, then the individual underweights the better option. This very general model—picked for its generality rather than its intrinsic appeal—includes as special cases popular ones such as those of probability weighting (Rank-Dependent Utility, Quiggin 1982, and Cumulative Prospect Theory, Tversky and Kahneman 1992) and Disappointment Aversion (Gul, 1991).[16] Moreover, it re-

[16]This specification also allows for generalizations of Rank-Dependent Expected Utility, e.g., the minimum from a set of probability distortions (Dean and Ortoleva, 2017). However, it does not

stricts preferences only for 50/50 lotteries, leaving complete freedom on the treatment of other lotteries.

Definition 3. We say that $\succsim$ admits a *Generalized Local Bilinear Utility* (GLBU) representation if there is a function $U : [w, b] \times T \to \mathbb{R}$ that represents preferences over deterministic outcomes satisfying Axioms 1-3[17] and a scalar $\pi(\frac{1}{2}) \in (0, 1)$, such that $p = \frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x',t')}$ with $U(x, t) \geq U(x', t')$ is evaluated according to:

$$V(p) = \pi(\frac{1}{2})U(x, t) + (1 - \pi(\frac{1}{2}))U(x', t').$$

To reiterate, a GLBU representation is a very general class that subsumes the vast majority of commonly used models. For time preferences, it allows for non-separable forms and non-exponential discounting; for risk, it allows for typical forms of non-Expected Utility.

Even in this very general class of models, violations of RSTL are still incompatible with Stochastic Impatience. As GLBU only restricts preferences over binary and equal-chance lotteries, we will focus on this type of lotteries.

Theorem 2. *Suppose $\succsim$ admits a GLBU representation. If $\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)}$ for some $x \in [w, b]$ and $t_1, t_2, t_3 \in T$ with $t_2 = \frac{t_1+t_3}{2}$, then $\succsim$ violates Stochastic Impatience.*

4 Two Solutions to Model RATL

We established a fundamental incompatibility between Stochastic Impatience and any violation of RSTL under the assumptions of Outcome Monotonicity, Impatience, No Future Bias, and Independence (or, more generally, Local Bilinearity). Given the appeal of Outcome Monotonicity, Impatience, and No Future Bias, we now explore how violations of RSTL can be accommodated by dropping one of the remaining properties. We show that it can be done by either (i) weakening Independence in a novel way, different from traditional ones and based on the intertemporal nature of our setup; or (ii) keeping full Independence but not requiring Stochastic Impatience, allowing a simple and tractable Expected Utility-type solution.

4.1 Drop 'Intertemporal' Independence

Our analysis in Section 3 assumes that individuals either evaluate risky prospects using Expected Utility, or follow one of its common generalizations via Local Bilinearity.

encompass all known models of risk preferences (e.g., the Cautious Expected Utility of Cerreia-Vioglio et al. 2015). Formally, this model is a local specification (at $\frac{1}{2}$) of the Bilinear (or Biseparable) model of Ghirardato and Marinacci (2001).

[17] That is, $U(x, t)$ is *i*) increasing in the first component, *ii*) decreasing in the second component, and *iii*) if $t < s$ and $\tau > 0$, then $U(x, t) = U(y, s)$ implies $U(x, t + \tau) \leq U(y, s + \tau)$.

Such generalizations, developed for a-temporal settings under risk and ambiguity, are based on the idea of underweighting or overweighting options to accommodate for phenomena like the certainty effect (Allais, 1953).

In an intertemporal setting, however, there are other context-specific relaxations of Independence. For example, we could require Independence to hold only for lotteries that pay prizes in the same period, but not for lotteries with prizes in different periods. One could argue that across-period comparisons involve a complex interaction of intertemporal tradeoffs that may render Independence less appealing. Formally, let $\succsim_t$ denote the restriction of $\succsim$ to lotteries that pay only in period t (i.e., with (y, s) in their support only if $s = t$). Then, we can impose 'within-period Independence': $\succsim_t$ satisfies Independence for each $t \in T$. While implied by general Independence, it is much weaker as it posits no restrictions across periods.

Such a relaxation of Independence is different from the ones implied by the Generalized Local Bilinear model. In the latter, individuals underweight (or overweight) the better prize and overweight (or underweight) the worst one, irrespectively of when each is paid. Thus they violate Independence within period *just as much* as across periods, as this general model has no specific distinctions for the intertemporal setup. In fact, with the restriction above the Generalized Local Bilinear model collapses to Expected Discounted Utility—if the individual does not distort probabilities within a period, she never does.

Intuitively, imposing Independence only within-period but not across periods would be useful precisely because it is the application of Independence *across* periods—even when weakened in the form of Local Bilinearity—that links Stochastic Impatience to RSTL. Without such a link, these two properties, that are obviously distinct, can be decoupled.

We illustrate how relaxing 'intertemporal' Independence allows for the joint presence of Stochastic Impatience and RATL with a simple example, derived from the finance literature and adapted to our setup: the Dynamic Ordinal Certainty Equivalent (DOCE) model (Selden, 1978; Selden and Stux, 1978). Preferences are represented by:

$$V(p) = \sum\nolimits_t \beta^t u\left(v^{-1}\left(\sum\nolimits_x \hat{p}_t(x) v(x)\right)\right), \tag{5}$$

where u and v are continuous and strictly increasing, $\beta \in (0, 1)$, and $\hat{p}_t(x) = p(x, t)$ for all (x, t) in the support of p and $\hat{p}_t(0) = 1 - \sum_{x \neq 0} p(x, t)$ for all t (throughout this section we assume that the prize 0, getting nothing, is available and that $u(0) = v(0) = 0$). The interpretation is that given a lottery p, the individual computes the marginal distribution over outcomes for any time period t, calculates the certainty equivalent of this induced lottery using the function v, and then aggregates these certainty equivalents as in EDU, with a discount function β and a utility function u. When no risk is involved, this model coincides with EDU (and thus automatically satisfies the monotonicity assumptions and No Future Bias). It satisfies Independence when lotteries pay in the same period, thus within-period Independence holds, but it

violates general Independence. The model is also not Bilinear.

Selden and Wei (2019) show that this model *always* satisfies Stochastic Impatience, and characterize when it is consistent with RATL.[18] It is easy to see why Stochastic Impatience must hold: both lotteries in the definition of Stochastic Impatience generate the same set of per-period certainty equivalents that are then discounted, and they only differ in the order these certainty equivalents are received; thus, higher values sooner must be preferred.[19] In fact, this model structurally disregards the across-period hedging that may lead to violations of Stochastic Impatience. To see why RSTL may fail, note that from the point of view of this model, the lottery that gives \$y with equal probability at either time t or t' induces an equal chance lottery at time t between \$0 and \$y, and an identical one at t'. If the individual is very risk averse, i.e., v is very concave, then both these lotteries have a low certainty equivalent, in particular lower than that of getting y for sure at time $\frac{t+t'}{2}$ (for which the curvature of v does not apply).

We should stress that this functional form is just *one example* of a model that can accommodate both violations of RSTL and Stochastic Impatience because it satisfies Independence only within—but not across—periods. A discussion of the appeal of this specific model is naturally beyond the scope of this paper.[20] The central message of this section, instead, is that one may be able to circumvent our impossibility result by weakening Independence in ways that are different from standard ones—which we have shown are not sufficient—, and are specific to the temporal setup.

A different way to see the content of this section is via the order in which time and risk are aggregated. In EDU, the order of aggregation does not matter. In all models considered before this section, the individual first aggregates time and then aggregates risk. For example, in equation (3), first each possible contingency (a realization (x, t)) is evaluated with a riskless utility that accounts for discounting; then, these utility-equivalents are aggregated taking the expectation (across contingencies). Such separability between mutually exclusive events is the one argued by Samuelson (1952) to motivate the Independence axiom. Indeed, this order is the one implied by the (full) Independence axiom itself: in any Expected Utility representation, the aggregation across contingencies must be the last, to guarantee linearity in probabilities. Weakening 'intertemporal' Independence allows one to switch the order of aggregation, first evaluating each time period's induced lottery and then aggregating those intertemporally—as in the DOCE representation considered above, where

[18] We thank Larry Selden for pointing out to us this functional form and these implications.

[19] Direct calculations show that if $x_1 > x_2$ and $t_1 < t_2$, then $V(\frac{1}{2}\delta_{(x_1,t_1)} + \frac{1}{2}\delta_{(x_2,t_2)}) - V(\frac{1}{2}\delta_{(x_2,t_1)} + \frac{1}{2}\delta_{(x_1,t_2)}) = (\beta^{t_1} - \beta^{t_2})\left(u\big(v^{-1}(\frac{1}{2}v(x_1))\big) - u\big(v^{-1}(\frac{1}{2}v(x_2))\big)\right) > 0$.

[20] Previous literature has highlighted how this model violates dynamic consistency and notions of stochastic monotonicity (Epstein and Zin, 1989; Chew and Epstein, 1990; Bommier et al., 2017). This approach also explicitly 'disregards' the correlation between outcomes in different times. For example, it treats the lottery that pays an extra x at time either t or t' with equal chances—a time lottery—identically to receiving either x or 0 with equal chance at both t and t' (with independent realizations).

the risk aggregation is done first within each time period and then the collection of certainty equivalents are aggregated as in discounted utility.[21]

4.2 Drop Stochastic Impatience, Maintain Expected Utility

An alternative way to model instances of violations of RSTL is by maintaining the full power of Independence, but no longer requiring Stochastic Impatience to hold. We now discuss a model that maintains the two core motivations behind EDU: exponentially discounted utility for deterministic payments and the evaluation of risk by "taking expectations." We show that, taken together, these two motivations do not lead to EDU. Rather, they lead to a general model that accommodates violations of RSTL; and indeed, RSTL is a key characterizing feature of the special case of EDU.

Generalized Expected Discounted Utility. We assume that the individual's behavior without risk follows standard exponentially discounted utility, i.e., $U(x,t) = \beta^t u(x)$ for some positive, continuous, and increasing function u over prizes and a discount factor $\beta \in (0,1)$. It is well-known (Fishburn and Rubinstein, 1982) that this functional form is characterized by Axioms 1-2 and strengthening Non Future Bias to Stationarity:

Axiom 3′ (Stationarity). *For all $x, y \in [w,b]$, $s,t \in T$, $\tau \in \mathbb{R}$ with $s+\tau, t+\tau \in T$, if $\delta_{(x,t)} \sim \delta_{(y,t+\tau)}$ then $\delta_{(x,s)} \sim \delta_{(y,s+\tau)}$.*

We also assume that, when facing non-degenerate lotteries, the individual uses the Expected Utility criterion—so his preferences satisfy Axioms 4 and 5. Putting together, we have the following result:

Proposition 1. *The following statements are equivalent:*

1. *$\succsim$ satisfies Axioms 1, 2, 3′, 4, 5;*
2. *There exist $\beta \in (0,1)$, $u : [w,b] \to \mathbb{R}_{++}$ and $\phi : \mathrm{Im}\big((\beta^{(\cdot)}u(\cdot)\big) \to \mathbb{R}$, both strictly increasing and continuous[22] such that $\succsim$ is represented by*

$$V(p) = \mathbb{E}_p(\phi(\beta^t u(x))).$$

We call this representation a *Generalized Expected Discounted Utility* model (GEDU) and identify it with the triple (u, β, ϕ). GEDU is similar to existing models in the literature. In particular, it can be seen as an application of the multi-attribute function of Kihlstrom and Mirman (1974) to the context of time.[23]

[21] The effect of the 'order of aggregation' was recently studied both theoretically (e.g., Apesteguia et al., 2019) and experimentally (Andreoni et al., 2017).

[22] $\mathrm{Im}((\beta^{(\cdot)}u(\cdot))$ is the image of $\beta^t u(x)$ over $[w,b] \times T$.

[23] A similar functional form was used, but not derived, by Andersen et al. (2017), to study intertemporal utility and correlation aversion, by Abdellaoui et al. (2017), to study different questions on time and risk, as well as by Edmans and Gabaix (2011) and Garrett and Pavan (2011).

Proposition 1 shows that combining the axioms that lead to exponential discounting without risk with the axioms that lead to Expected Utility does not generate EDU, but a model that includes one additional curvature, captured by the function ϕ, applied *after* discounting has taken place. The model only coincides with EDU when ϕ is affine. The proposition follows immediately from standard arguments: it is a consequence of the fact that one cannot assume that the Bernoulli utility used in the Expected Utility form is, cardinally, the discounted utility.[24]

Under EDU, time and risk preferences are both governed solely by the curvature of u. This is no longer the case for GEDU. Intertemporal substitution is governed by u and β: without risk, the individual evaluates a prize x payed at time t by $\beta^t u(x)$ (ϕ, being strictly increasing, won't matter). Atemporal risk preferences, for lotteries with only immediate payments, are instead governed by $\phi \circ u$: a lottery p that pays only at time 0 is evaluated by $\mathbb{E}_p(\phi(u(x)))$. Thus, under GEDU, intertemporal substitution and risk aversion differ—the difference captured by the curvature of ϕ. One possible interpretation is that u represents the individual's utility function over deterministic payments, while ϕ represents risk attitude towards variations in 'discounted utils.' As we now show, unlike EDU, GEDU does not constrain preferences to be RSTL:

Proposition 2. *Consider $\succsim$ that admits a GEDU representation (u, β, ϕ). Then:*

1. *$\succsim$ is RSTL if and only if ϕ is a convex transformation of* ln*;*
2. *$\succsim$ is RNTL if and only if ϕ is an affine transformation of* ln*;*
3. *$\succsim$ is RATL if and only if ϕ is a concave transformation of* ln*.*

To understand Proposition 2, note that if $\phi = \ln$, then $\phi(\beta^t u(x)) = t \ln \beta + \ln u(x)$, an affine function of t, implying RNTL. If ϕ is "more concave than the log," preferences are RATL; if it is "more convex than the log," preferences are RSTL. Note also that increasing the curvature of ϕ generates a higher risk aversion towards both time and monetary prizes. As discussed in Section 2, the connection between these two forms of risk aversion is supported by our experimental results.

Risk Stationarity and what is missing for EDU. In characterizing GEDU, we imposed Stationarity only on trade-offs involving deterministic payments (Axiom 3′). Consider a strengthening to risky prospects: the ranking between two lotteries should not change if we move all payments in the support of the lotteries by the same number of periods. Formally, for any $p \in \Delta$, let $p_{+\tau}$ denote the lottery in which each prize is shifted by τ periods: $p_{+\tau}((x, t+\tau)) = p((x, t))$ for all $(x, t) \in [w, b] \times T$.

[24] Fishburn and Rubinstein (1982) show that Axioms 1-3′ imply a Discounted Utility representation for degenerate lotteries, $\beta^t u(x)$. By Axioms 4 and 5, preferences over lotteries follow Expected Utility using Bernoulli utility v, which must be *ordinally*, but not necessarily *cardinally*, equivalent to $\beta^t u(x)$. Thus, there must exist a strictly increasing function ϕ such that $v(x, t) = \phi(\beta^t u(x))$.

Axiom 3″ (Risk Stationarity). *For every $p, q \in \Delta$ and τ such that $p_{+\tau}, q_{+\tau} \in T$,*

$$p \succsim q \;\Leftrightarrow\; p_{+\tau} \succsim q_{+\tau}.$$

As usual, we say that $\succsim$ is *strictly RSTL* if any non-degenerate time lottery is strictly preferred to receiving the prize at the expected time. Note that EDU satisfies both strict RSTL and Risk Stationarity.[25] Our next result establishes that, within GEDU, the converse is also true. That is, starting from the properties of exponentially discounted utility and Expected Utility, EDU is characterized by imposing both Risk Stationarity and RSTL.[26]

Proposition 3. *Suppose T is an interval and consider $\succsim$ that satisfies Axioms 1, 2, 3″, 4, and 5 (so that it admits a GEDU representation that satisfies Risk Stationarity). Then, $\succsim$ is strictly RSTL if and only if it admits an EDU representation.*

RATL and Stochastic Impatience under GEDU. We conclude by connecting GEDU to our previous impossibility result. In this context, Stochastic Impatience not only implies RSTL (as must be the case by Theorem 1), but the converse is also true.

Proposition 4. *Suppose that $\succsim$ admits a GEDU representation. The following statements are equivalent:*

1. *The relation $\succsim$ satisfies Stochastic Impatience;*
2. *The relation $\succsim$ is RSTL.*

Recall that, under Expected Utility, Stochastic Impatience is equivalent to $\phi(\beta^t u(x))$ having decreasing differences (see equation 4). To understand Proposition 4, suppose that T is an interval, ϕ is twice differentiable, and u is differentiable.[27] Then, decreasing differences holds if and only if $\frac{\partial}{\partial x \partial t}\left[\phi\left(\beta^t u\left(x\right)\right)\right] \leq 0$. Let $\phi = g \circ \ln$ for some increasing function g. Calculating the cross-partial derivative, we find that this condition holds if and only if g is convex. Therefore, preferences satisfy Stochastic Impatience if and only if ϕ is more convex than ln, which, as shown in Proposition 2, is also the condition for preferences to be RSTL.

[25] The normative appeal of Risk Stationarity, linked to dynamic consistency, is to be contrasted with robust evidence of its violations. For example, subjects are typically more risk tolerant for delayed payments (see Abdellaoui et al. 2011 and many references therein), which is compatible with GEDU if ϕ exhibits (strictly) increasing relative risk aversion.

[26] There are GEDU preferences that allow for RATL while maintaining Risk Stationarity. It can be shown that these coincide with a 'Negative-EDU' model, where $\beta > 1$ and the utility u is negative (e.g., CRRA more concave than the *log*; a negative utility guarantees that Impatience holds even though $\beta > 1$). While this may be a tractable model, this particular result does not carry over to lotteries over streams of consumption.

[27] We thank Rakesh Vohra for suggesting this connection.

5 Conclusion

This paper studies attitudes towards lotteries in which both the prize and the payment date are uncertain, and their interactions with other aspects of risk and time preferences. We introduce two notions: (i) risk attitude towards time lotteries, which governs how an individual treats lotteries with uncertain timing, and (ii) Stochastic Impatience, a stochastic counterpart of the standard Impatience axiom.

We show in an incentivized experiment that subjects are not consistently risk seeking about the date of payment (RSTL), in contrast to the predictions of EDU. The paper then makes two theoretical contributions.

First, we provide an impossibility result on modeling individuals that are not globally RSTL. Under mild assumptions, satisfied by most generalizations of EDU (including most forms of non-exponential discounting and non-Expected Utility), any violation of RSTL implies that the individual must also violate Stochastic Impatience.

Second, we suggest two ways to accommodate violations of RSTL. First, we show that a suitable relaxation of Independence, which does not require it to hold intertemporally, relaxes the impossibility above and allows for both violations of RSTL and Stochastic Impatience. In the second approach, we show that if one does not require Stochastic Impatience to hold, then violations of RSTL can be accommodated while preserving the main behavioral features of EDU.

Conceptually, this paper suggests two new behavioral notions pertaining to the economics of risk and time, that can be used to assess the validity of existing models. In general, one needs to use their subjective judgment to decide to which of them to (globally) adhere. What is common to both properties is that they make it clear why, within the standard model of EDU, it is not innocuous to use the same parameters elicited in a risk free environment also in the presence of risk: RSTL translates diminishing willingness to wait into risk loving with respect the time in which a good is received; Stochastic Impatience further shows how the simple notion of discounting implies risk seeking towards 'discounted utils.'

Throughout the paper, we considered lotteries over dated rewards (x, t) for expositional purposes. In Appendix A, we show that both our impossibility results generalize to preferences on lotteries over consumption streams. Similarly, both our proposed solutions generalize. The discussion in Section 4.1 directly extends. The extension of GEDU to this case replaces $\phi(\beta^t u(x))$ with $\phi(\sum_{t=0}^{\infty} \beta^t u(x_t))$. This version also allows for RATL, but when it does it also violates Stochastic Impatience. The latter functional form, however, is not dynamically consistent (unless it coincides with EDU). One may instead consider the model of Epstein and Zin (1989), defined on a much richer domain of temporal lotteries, related to GEDU but dynamically consistent. In Appendix B we consider its widely-used CRRA-CES version, and show that it is capable of accommodating violations of RSTL—when risk aversion is sufficiently greater than the inverse of the elasticity of intertemporal substitution. However, just as with GEDU, whenever RATL is allowed, Stochastic Impatience is

violated, showing that the main point of our impossibility result applies to the model of Epstein and Zin as well.[28]

[28]Dillenberger et al. (2018) discuss the relation between models that separate time and risk preferences (such as Epstein and Zin, 1989) and GEDU for preferences over streams, and show that Stochastic Impatience imposes a bound on the individual's risk aversion holding intertemporal substitution fixed.

Appendices

These appendices contain some extensions (in Appendices A and B), the proofs of all results except those of Appendix B (in Appendix C), and additional information about our experiments (in Appendix D). The proofs of results in Appendix B and the questionnaires and screenshots of our experiments are in the Supplementary Appendix.

A Extension to Lotteries Over Consumption Streams

We now extend our results to lotteries over consumption streams. We focus on discrete time; the case of continuous time is analogous.

Setup. Let $[w, b] \subseteq \mathbb{R}_{++}$ be an interval of prizes and let $T = 0, 1, 2, ...$ be the set of dates. A consumption stream is an element of $[w, b]^T$ and our primitive is a complete and transitive preference relation $\succsim'$ over simple lotteries on consumption streams, $\Delta([w, b]^T)$. For convenience, let $[c, (x, t)]$ denote the consumption stream that returns $c \in [w, b]$ in every period ("background consumption") except $c + x$ in period t, i.e., $\left(c, c, ..., \underbrace{c + x}_{t}, c, ...\right)$, where $c + x \in [w, b]$. (For brevity, we will omit the requirement that $c + x \in [w, b]$ in the definitions below.) We impose the analogous of the richness condition for the space of consumption streams, requiring $\delta_{[c,(b-c,t+1)]} \succ' \delta_{[c,(w-c,t-1)]}$ for any $t > 0$ and any c.

A time lottery $p_{c,x}$ with prize x is a lottery over streams $[c, (x_i, t_i))]$ in which all streams have the same background consumption c and the addition x is the same in any possible realization, that is, $x_i = x$ for all i.

Our notions of RSTL and Stochastic Impatience read in this domain as follows:

Definition 4. The relation $\succsim'$ is Risk Seeking over Time Lotteries (RSTL) if for all time lotteries $p_{c,x}$ with mean date t,

$$p_{c,x} \succsim' \delta_{[c,(x,t)]}$$

for all $c \in [w, b]$ and $x > 0$.

Definition 5. We say that $\succsim'$ satisfies Stochastic Impatience if for any $c \in [w, b]$, $t_1, t_2 \in T$ with $t_1 < t_2$, and $x_1 > x_2 > 0$,

$$\frac{1}{2}\delta_{[c,(x_1,t_1)]} + \frac{1}{2}\delta_{[c,(x_2,t_2)]} \succsim' \frac{1}{2}\delta_{[c,(x_1,t_2)]} + \frac{1}{2}\delta_{[c,(x_2,t_1)]}.$$

In extending Theorem 1 and Theorem 2, we first translate Axioms 1-3 into this new domain. For brevity, we impose them directly on the utility function over streams

$U : [w, b]^T \to \mathbb{R}$; the corresponding properties on $\succsim'$ are immediately derived. While in applications one may want to impose stronger assumptions, for our purposes it suffices to impose conditions on how streams of type $[c, (x, t))]$ are evaluated. We assume (i) Outcome Monotonicity: For all $c \in [w, b]$ and $s \in T$, if $x > y$ then $U([c, (x, s)]) > U([c, (y, s)])$; (ii) Impatience: For all $c \in [w, b]$, $x > 0$, and $s, t \in T$, if $t < s$ then $U([c, (x, t)]) > U([c, (x, s)])$; and (iii) No Future Bias: For all $c \in [w, b]$, $s, t \in T$ with $t < s$, and $\tau > 0$ with $s + \tau, t + \tau \in T$, if $U([c, (x, t)]) = U([c, (y, s)])$ then $U([c, (x, t + \tau)]) \leq U([c, (y, s + \tau)])$.

Impossibility results. We first derive the results for Expected Utility. Suppose preferences are represented by

$$V(p) = \mathbb{E}_p\big[U(x_0, x_1, ...)\big] \tag{6}$$

for some continuous function $U : [w, b]^T \to \mathbb{R}$ satisfying assumptions (i)-(iii).

THEOREM 3. *Suppose $\succsim'$ admits a representation as in equation (6). Then Stochastic Impatience implies RSTL.*

We now extend the results to preferences that admit a Generalized Local Bilinear representation:

$$V\left(\frac{1}{2}\delta_{(x_0,x_1,\dots)} + \frac{1}{2}\delta_{(x'_0,x'_1,\dots)}\right) = \phi\left(\frac{1}{2}\right) U\left(x_0, x_1, ...\right) + \left[1 - \phi\left(\frac{1}{2}\right)\right] U\left(x'_0, x'_1, ...\right) \tag{7}$$

whenever $U\left(x_0, x_1, ...\right) \geq U\left(x'_0, x'_1, ...\right)$, for some $\phi(\frac{1}{2}) \in (0, 1)$ and some continuous function $U : [w, b]^T \to \mathbb{R}$ satisfying assumptions (i)-(iii).

THEOREM 4. *Suppose $\succsim'$ admits a representation as in equation (7). If $\delta_{[c,(x,t_2)]} \succ' \frac{1}{2}\delta_{[c,(x,t_1)]} + \frac{1}{2}\delta_{[c,(x,t_3)]}$ for some $x \in [w, b]$, $t_1, t_2, t_3 \in T$ with $t_2 = \frac{t_1+t_3}{2}$, then $\succsim$ violates Stochastic Impatience.*

The proofs of the previous two theorems appear in Appendix C. They follow the exact same steps of the proofs of the corresponding theorems in the main text, after replacing each term $U(x, t)$ with $U([c, (x, t)])$.

Solutions to model RATL. We now discuss how our solutions to model RATL presented in Section 4 extend to this setup. The discussion in Section 4.1 generalizes immediately; as an example, one could use the original DOCE model (Selden, 1978; Selden and Stux, 1978); see also Selden and Wei (2019) for further discussion.

We can also easily extend the GEDU representation to this domain. It will now be composed of a continuous and strictly increasing $u : [w, b] \to \mathbb{R}$, $\beta \in (0, 1)$, and a strictly increasing $\phi : \mathbb{R}_+ \to \mathbb{R}$, such that $\succsim'$ is represented by

$$V(p) = \mathbb{E}_p\left[\phi\left(\sum_{t=0}^{\infty} \beta^t u(x_t)\right)\right]. \tag{8}$$

This functional form is the one of Kihlstrom and Mirman (1974) applied to our setting. It can be characterized, like GEDU, by imposing the axioms for Discounted Utility and Expected Utility. It is easy to see how this model can accommodate RATL. If ϕ is concave enough the value of the non-degenerate lottery gets arbitrary close to the value of the worst element in the support (the one in which the prize x is received in the latest possible period); against it, the individual would prefer to receive x in the average time $\bar{t}$.

From Theorem 3, whenever this model violates RSTL, it must also violate Stochastic Impatience. In fact, following the same argument as in Proposition 4, it can be shown that this model satisfies Stochastic Impatience if and only if preferences are RSTL.

It is known, however, that if applied recursively this model is not dynamically consistent. One solution is thus to adopt the model of Epstein and Zin (1989), discussed below, which allows for RATL while mainting all other properties—except violating Stochastic Impatience whenever RATL is permitted.

B Epstein-Zin Preferences

We now show that the model of Epstein and Zin (1989) (henceforth, EZ) with CRRA Expected Utility preferences and CES aggregator can accommodate violations of RSTL. When it does, however, it also violates Stochastic Impatience.

We adopt the same recursive setup as in their paper, which we do not discuss here for brevity. Consider a preference relation $\hat{\succsim}$ that admits a recursive representation of the form:

$$V_t = \left\{ (1-\beta)\, C_t^{1-\rho} + \beta \left[E_t \left(V_{t+1}^{1-\alpha} \right) \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}, \tag{9}$$

where C_t denotes consumption at time t, $\alpha \geq 0$ is the coefficient of relative risk aversion, and $\rho \geq 0$ is the inverse of the elasticity of intertemporal substitution (with $\alpha \neq 1$ and $\rho \neq 1$, so that the formula is well-defined). EZ coincides with EDU when $\alpha = \rho$.

Since a degenerate object in this model is a consumption stream (rather than a dated reward), we use the definitions of Stochastic Impatience and RATL as introduced in Appendix A. In this setup we also need to specify when the uncertainty is resolved: for all the lotteries in question, we assume for simplicity that the uncertainty is resolved immediately after the current period.

We first show that EZ allows for violations of RSTL although it cannot accommodate (global) RATL:

Proposition 5. *Under EZ, for any β, ρ, and x, there exists $\bar{\alpha}_{\rho,\beta,x} > \max\{\rho, 1\}$ such that $\delta_{(x,t)} \hat{\succ} \frac{1}{2}\delta_{(x,t-1)} + \frac{1}{2}\delta_{(x,t+1)}$ if and only if $\alpha > \bar{\alpha}_{\rho,\beta,x}$. Moreover, $\lim_{x \searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$.*

Proposition 5 shows that, controlling for discounting β, elasticity of intertemporal substitution $1/\rho$, and the size of the prize x, more risk averse individuals are more likely to prefer the safe lottery and violate RSTL.[29] That is, as with GEDU, there is also a connection between risk aversion over time lotteries and risk aversion over temporal lotteries in EZ. Moreover, the risky time lottery is always preferred if the utility function is less concave than a logarithmic function ($\alpha < 1$) and if $\alpha \leq \rho$.[30] Since $\lim_{x \searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$, it will also be preferred if the prize x is small enough. That is, EZ preferences cannot be (globally) RATL.

Also this model, however, cannot accommodate violations of RSTL without also violating Stochastic Impatience:

Proposition 6. *Suppose that $\hat{\succsim}$ admits an EZ representation. Then, if $\hat{\succsim}$ satisfies Stochastic Impatience, it also satisfies RSTL.*

This result shows that the impossibility of accommodating violations of RSTL while preserving Stochastic Impatience (Theorem 1) carries over to this richer setting. The intuition is very similar to the one previously given: the extra "intertemporal" risk aversion needed to accommodate RATL is going to generate a violation of Stochastic Impatience. We refer to Dillenberger et al. (2018) for an in-depth discussion of the implications of Stochastic Impatience for EZ.

C Proofs

C.1 Proof of Theorem 1

The proof proceeds as follows. In Step 1, we prove that any violation of RSTL implies a violation of RSTL with binary equally-likely lotteries. In Step 2, we prove that any violation of RSTL with binary equally-likely lotteries implies a violation of RSTL with binary equally-likely lotteries in which dates are either consecutive, in the case of discrete time, or are arbitrarily close, in the case of continuous time. Lastly, Step 3 proves that any such violation of RSTL implies a violation of Stochastic Impatience.

Step 1. We show that RSTL with respect to binary, equally-likely lotteries implies RSTL with respect to arbitrary lotteries. Fix a prize x and let p_x be a time lottery with prize x, where for all t, $p(x,t)$ is a dyadic rational. For RSTL to apply, it must be that $\sum_t p(x,t)t \in T$. Let

$$n^* := \max \left\{ n : p(x,t) = a \times 2^{-n} \text{ for some } (x,t) \text{ in the support of } p_x \text{ and } a \in \mathbb{Z} \right\}.$$

[29] That is, $\alpha' > \alpha$ implies that if the decision maker with coefficient of risk aversion α prefers the safe lottery over the risky one, so does the decision maker with α' (holding other parameters fixed).

[30] Starting with Kreps and Porteus (1978), a large literature has studied preferences over the timing of resolution of uncertainty. With EZ, early resolution of uncertainty is preferred if and only if $\alpha > \rho$ (Epstein et al., 2014). Proposition 5 then implies that this condition is also needed for the safe time lottery to be preferred.

Since p_x is finite, n^* is well defined. Rewrite p_x as a lottery p_x^0 that gives all prizes with probabilities 2^{-n^*} each. Note that each (x,t) in the support of p_x^0 appears now $a(t)$ times, where $a(t) = \frac{p(x,t)}{2^{n^*}}$. Proceed inductively. The key observation is that we can always group terms to rewrite p_x^0 as a lottery p_x^1 which consists of 2^{n^*-1} lotteries of the form $\frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x,t')}$, with $\frac{t+t'}{2} \in T$, each is obtained with probability 2^{1-n^*} (note that t and t' needn't be distinct). By Independence, the value of p_x^1 weakly decreases if we repeatedly replace each $\frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x,t')}$ with a less preferred lottery. And by binary RSTL, we know that $\frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x,t')} \succsim \delta_{(x,\frac{t+t'}{2})}$. Replace then all of those sublotteries with $\delta_{(x,\frac{t+t'}{2})}$. For any $k \leq n^*$, proceed similarly to obtain p_x^k which consists of 2^{n^*-k} lotteries of the form $\frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x,t')}$, with $\frac{t+t'}{2} \in T$, each is obtained with probability 2^{k-n^*}, and repeatedly apply Independence and binary RSTL as above. When $k = n^*$, replace the last term $\frac{1}{2}\delta_{(x,t)} + \frac{1}{2}\delta_{(x,t')}$ with $\delta_{(x,\frac{t+t'}{2})}$. By transitivity, $p_x \succsim \delta_{(x,\frac{t+t'}{2})}$. Since x was arbitrary and since the set of dyadic rationals is dense in $\mathbb{R}$, continuity implies that RSTL applies for *any* time lottery.

Step 2. We now show that a violation of RSTL in binary, equally-likely lotteries, implies that we can construct a violation with binary, equally-likely lotteries in which we either have consecutive dates, with discrete time; or the dates can be arbitrarily close, in the case of continuous time. In particular, in either case the dates are close enough that our richness condition applies.

Claim 1. *Let $\{L,H\} \in T$ with $L < H$ and $y \in X$ such that*

$$U\left(y, \frac{H+L}{2}\right) > \frac{U(y,H) + U(y,L)}{2}.$$

Then, there exists $t_1, t_2, t_3 \in T$ with $t_1 < t_2 < t_3$, $\frac{t_1+t_3}{2} = t_2$, $\delta_{(b,t_3)} \succ \delta_{(w,t_1)}$, and

$$U(y,t_2) > \frac{U(y,t_1) + U(y,t_3)}{2}.$$

Proof. Note that

$$U\left(y, \frac{H+L}{2}\right) > \frac{U(y,H) + U(y,L)}{2}$$

$$\iff U\left(y, \frac{H+L}{2}\right) - U(y,L) > U(y,H) - U\left(y, \frac{H+L}{2}\right).$$

Suppose first that T is discrete. Let $\Delta_t := U(y,t) - U(y,t-1)$. If the claim is false, we must have Δ_t decreasing in t for all $t \in \{L+1, ..., H\}$, since

$$U(y,t) \leq \frac{U(y,t+1) + U(y,t-1)}{2} \iff \Delta_t = U(y,t) - U(y,t-1) \leq U(y,t+1) - U(y,t) = \Delta_{t+1}.$$

But

$$U\left(y, \frac{H+L}{2}\right) - U(y,L) = \sum_{t=L+1}^{\frac{H+L}{2}} \Delta_t \geq \sum_{t=\frac{H+L}{2}}^{H} \Delta_t = U(y,H) - U\left(y, \frac{H+L}{2}\right),$$

where the inequality uses the fact that Δ_t is decreasing in t and that $\frac{H+L}{2}$ is the midpoint between H and L (so the summation has the same number of elements). But this is a contradiction.

Now suppose that T is continuous. Consider $\epsilon \in \mathbb{R}_+$ small enough such that $\delta_{(b,t+\epsilon)} \succ \delta_{(w,t-\epsilon)}$ for all $t \in [L+\epsilon, H-\epsilon]$, the existence of which is implied by continuity. The proof for this case is then identical to the one above, simply replacing $t+1$ and $t-1$ with $t+\epsilon$ and $t-\epsilon$. ‖

Step 3. We now prove that a violation of RSTL as those obtained in Claim 1 imply a violation of Stochastic Impatience. Suppose, by contradiction, that there are x and $t_1 < t_2 < t_3$ with $\frac{t_1+t_3}{2} = t_2$, such that $\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)}$. The assumption that $\delta_{(b,t_3)} \succ \delta_{(w,t_1)}$ guarantees that either (i) there is $x' < x$ such that $\delta_{(x',t_1)} \sim \delta_{(x,t_2)}$, or (ii) there is $x' > x$ such that $\delta_{(x',t_3)} \sim \delta_{(x,t_2)}$, or both.

Suppose case (i) holds. Then $\delta_{(x',t_1)} \sim \delta_{(x,t_2)}$ and by No Future Bias $\delta_{(x',t_2)} \precsim \delta_{(x,t_3)}$. By Independence and by the assumption above, $\frac{1}{2}\delta_{(x',t_1)} + \frac{1}{2}\delta_{(x,t_2)} \sim \delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)} \succsim \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x',t_2)}$. By transitivity, $\frac{1}{2}\delta_{(x',t_1)} + \frac{1}{2}\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x',t_2)}$, violating Stochastic Impatience.

Suppose case (ii) holds. Then $\delta_{(x',t_3)} \sim \delta_{(x,t_2)}$ and by No Future Bias $\delta_{(x',t_2)} \precsim \delta_{(x,t_1)}$. By Independence and by the assumption above, $\frac{1}{2}\delta_{(x',t_3)} + \frac{1}{2}\delta_{(x,t_2)} \sim \delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)} \succsim \frac{1}{2}\delta_{(x',t_2)} + \frac{1}{2}\delta_{(x,t_3)}$. By transitivity, $\frac{1}{2}\delta_{(x',t_3)} + \frac{1}{2}\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x',t_2)} + \frac{1}{2}\delta_{(x,t_3)}$, violating Stochastic Impatience. ■

C.2 Proof of Theorem 2

Consider some $x \in X$ and $t'_1, t'_2, t'_3 \in T$ with $t'_1 < t'_2 < t'_3$, $t'_2 = \frac{t'_1+t'_3}{2}$ such that $\delta_{(x,t'_2)} \succ \frac{1}{2}\delta_{(x,t'_1)} + \frac{1}{2}\delta_{(x,t'_3)}$. By Claim 1 in the proof of Theorem 1, we then there must also be $t_1, t_2, t_3 \in T$ with $t_1 < t_2 < t_3$, $t'_2 = \frac{t_1+t_3}{2}$ such that not only $\delta_{(x,t_2)} \succ \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)}$ but we also have $\delta_{(b,t_3)} \succ \delta_{(w,t_1)}$. Denote $r_x := \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x,t_3)}$. According to the GLBU representation we have

$$V(\delta_{(x,t_2)}) = U(x, t_2)$$

and

$$V(r_x) = \pi(\frac{1}{2})U(x, t_1) + (1 - \pi(\frac{1}{2}))U(x, t_3).$$

Let $\overline{\pi}$ be the value such that $V(\delta_{(x,t_2)}) = V(r_x)$, i.e.,

$$\overline{\pi} = \frac{U(x, t_2) - U(x, t_3)}{U(x, t_1) - U(x, t_3)} \in (0, 1).$$

The richness condition $U(w, t_1) < U(b, t_3)$ guarantees that either (i) there is $x' < x$ such that $U(x', t_1) = U(x, t_2)$; or (ii) there is $x' > x$ such that $U(x', t_3) = U(x, t_2)$; or both.

Consider first case (i). Take $x' < x$ such that $U(x', t_1) = U(x, t_2)$. Let $p = \frac{1}{2}\delta_{(x,t_1)} + \frac{1}{2}\delta_{(x',t_2)}$ and $q = 0.5\delta_{(x',t_1)} + \frac{1}{2}\delta_{(x,t_2)}$. We have

$$V(p) = \pi(\frac{1}{2})U(x, t_1) + (1 - \pi(\frac{1}{2}))U(x', t_2)$$

and

$$V(q) = U(x, t_2).$$

Let $\widehat{\pi}$ be the value such that $V(p) = V(q)$, or

$$\widehat{\pi} = \frac{U(x, t_2) - U(x', t_2)}{U(x, t_1) - U(x', t_2)} \in (0, 1).$$

Note that $\pi(\frac{1}{2}) > \widehat{\pi}$ implies $V(p) > V(q)$, while $\pi(\frac{1}{2}) < \overline{\pi}$ implies $V(\delta_{(x,t_2)}) > V(r)$. We will be done if we show that $U(x', t_2) \leq U(x, t_3)$, since this implies that $\widehat{\pi} \geq \overline{\pi}$, so that Stochastic Impatience and RATL contradict one another. But this follows by No Future Bias: since $t_2 - t_1 = t_3 - t_2$, $U(x', t_1) = U(x, t_2)$ implies $U(x', t_1 + (t_2 - t_1)) \leq U(x, t_2 + (t_3 - t_2))$, thus $U(x', t_2) \leq U(x, t_3)$.

If case (i) is not satisfied, then consider now case (ii). Take $x' > x$ such that $U(x', t_3) = U(x, t_2)$. Let $p = \frac{1}{2}\delta_{(x',t_2)} + \frac{1}{2}\delta_{(x,t_3)}$ and $q = \frac{1}{2}\delta_{(x,t_2)} + \frac{1}{2}\delta_{(x',t_3)}$. We have

$$V(p) = \pi(\frac{1}{2})U(x', t_2) + (1 - \pi(\frac{1}{2}))U(x, t_3)$$

and

$$V(q) = U(x, t_2).$$

Let $\widehat{\pi}$ be the value such that $V(p) = V(q)$, or

$$\widehat{\pi} = \frac{U(x, t_2) - U(x, t_3)}{U(x', t_2) - U(x, t_3)} \in (0, 1).$$

If $U(x', t_2) \leq U(x, t_1)$ then we have $\widehat{\pi} \geq \overline{\pi}$. But by No Future Bias (applied "backwards"), and since $t_2 - t_1 = t_3 - t_2$, $U(x', t_3) = U(x, t_2)$ implies $U(x', t_3 - (t_3 - t_2)) \leq U(x, t_2 - (t_2 - t_1))$, or $U(x', t_2) \leq U(x, t_1)$. ■

C.3 Proof of Proposition 1

That (2) implies (1) is immediate. For the other direction, note that by Continuity, for all $(x, t) \in [w, b] \times T$ the sets $\{(x, t) \in [w, b] \times T : \delta_{(x,t)} \succsim \delta_{(y,s)}\}$ and $\{(x, t) \in [w, b] \times T : \delta_{(y,s)} \succsim \delta_{(x,t)}\}$ are closed in the product topology on $[w, b] \times T$. Define $\succsim'$ on $[w, b] \times T$ by $(x, s) \succsim' (y, t)$ if and only if $\delta_{(x,s)} \succsim \delta_{(y,t)}$, and note that $\succsim'$ satisfies Axioms A0-A5 in Fishburn and Rubinstein (1982). Then, by Theorem 2 in that

paper, there exist $\beta \in (0,1)$ and a strictly increasing and continuous $u : [w,b] \to \mathbb{R}_{++}$ such that

$$\delta_{(x,s)} \succsim \delta_{(y,t)} \;\Leftrightarrow\; (x,s) \succsim' (y,t) \;\Leftrightarrow\; \beta^s u(y) \geq \beta^t u(x).^{31}$$

By Independence and Continuity there exists $U : [w,b] \times T \to \mathbb{R}$ such that

$$p \succsim q \;\Leftrightarrow\; \mathbb{E}_p(U) \geq \mathbb{E}_p(U).$$

By Continuity, U is also continuous.

It follows that for all $(x,s),(y,t) \in [w,b] \times T$, $\beta^s u(x) \geq \beta^t u(y)$ if and only if $U(x,s) \geq U(y,t)$. Let $F(x,t) = \beta^t u(x)$. The existence of $\phi : F([w,b] \times T) \to \mathbb{R}$ such that $U(x,t) = \phi(F(x,t))$ follows from standard arguments, as both U and F represent the same preferences. The continuity of ϕ is also immediate. We are left with showing that such ϕ is strictly increasing. If not, then there exist $a,b \in F([w,b] \times T)$ such that $a > b$ but $\phi(a) = \phi(b)$. Since $a,b \in F([w,b] \times T)$, there exist $(x,t),(y,s) \in [w,b] \times T$ such that $F(x,t) = a > b = F(y,s)$, thus $\delta_{(x,t)} \succ \delta_{(y,s)}$. But since $\phi(a) = \phi(b)$ we have $U(x,t) = U(y,s)$, thus $\delta_{(x,t)} \sim \delta_{(y,s)}$, a contradiction. ■

C.4 Proof of Proposition 2

Let r_x be a time lottery which yields x in a random time t with $E_r(t) = \bar{t}$. Then

$$V(\delta_{(x,\bar{t})}) = \phi\left(\beta^{\bar{t}} u(x)\right)$$

and

$$V(r_x) = E_r \phi\left(\beta^t u(x)\right).$$

Note that if $\phi = \ln$, then $V(\delta_{(x,\bar{t})}) = V(r_x) = \bar{t} \ln \beta + \ln u(x)$. Since the distribution of t is a mean-preserving spread of the distribution of $\bar{t}$, Jensen inequality implies that $V(\delta_{(x,\bar{t})}) \geq$ (resp., $\leq$) $V(r_x)$ whenever there is a concave (resp., convex) function h such that $\phi = h \circ \log$.

Since r_x was arbitrary and the domain of ϕ, $\mathrm{Im}\big((\beta^{(\cdot)} u(\cdot)\big)$, is an interval, the concavity (resp., convexity) of h needs to hold everywhere to ensure no violation of RATL (resp., RSTL). ■

C.5 Proof of Proposition 3

It is immediate that EDU satisfies Risk Stationarity and is strictly RSTL. We need to show that any preference relation that admits a GEDU representation, satisfies Risk Stationarity, and is strictly RSTL has an EDU representation. The proof will use the following result:

[31] Recall that our domain includes only strictly positive prizes, so that we do not add the requirements for u on weakly negative outcomes.

Claim 2. *Let T be an interval. Consider $\succsim$ that admits a GEDU representation (u, β, ϕ). Then, $\succsim$ satisfies Risk Stationarity if and only if ϕ has constant relative risk aversion.*

Proof. To simplify notation, let $\bar{U} \equiv \mathrm{Im}((\beta^{(\cdot)}u(\cdot))$ denote the domain of ϕ. Suppose $\phi : \bar{U} \to \mathbb{R}$ does not have constant relative risk aversion. Then, there must exist a simple lottery p with outcomes $v_i \in \bar{U}$ and expected value $\bar{p} \equiv \sum p_i v_i$, $a \in (0,1)$ such that $av_i \in \bar{U}$ for all i, and $\bar{z} \in \bar{U}$ such that

$$\sum p_i \phi(v_i) = \phi(\bar{z}) \quad \text{but} \quad \sum p_i \phi(av_i) \neq \phi(a\bar{z}).$$

Let

$$\bar{a} \equiv \sup\{a \in (0,1) : \sum p_i \phi(av_i) \neq \phi(a\bar{z})\}.$$

There are two cases: $\bar{a} < 1$ and $\bar{a} = 1$.

Consider first the case of $\bar{a} < 1$. By the definition of the supremum, $\sum p_i \phi(av_i) = \phi(a\bar{z})$ for all $a \in (\bar{a}, 1]$ and, for any $\epsilon > 0$, there exists $a'_\epsilon \in (\bar{a} - \epsilon, \bar{a})$ with $\sum p_i \phi(av_i) \neq \phi(a\bar{z})$. Take $\tau_\epsilon \in T$ with

$$\beta^{\tau_\epsilon} = \frac{a'_\epsilon}{\bar{a}},$$

which exists when ϵ is small enough since $\lim_{\epsilon \searrow 0} \frac{a'_\epsilon}{\bar{a}} = 1$. Then, it follows that $\tilde{p}_{+\tau_\epsilon} \nsim \tilde{q}_{+\tau_\epsilon}$, contradicting Risk Stationarity.

Next, consider the case of $\bar{a} = 1$. By the definition of the supremum, for any $\epsilon > 0$, there exists $a'_\epsilon \in (\bar{a} - \epsilon, 1)$ with $\sum p_i \phi(av_i) \neq \phi(a\bar{z})$. Take

$$\beta^{\tau_\epsilon} = a'_\epsilon,$$

which again exists when ϵ is small enough since $\lim_{\epsilon \searrow 0} a'_\epsilon = 1$. Then, it follows that

$$\tilde{p}_{+\tau_\epsilon} \nsim \tilde{q}_{+\tau_\epsilon},$$

contradicting Risk Stationarity. ‖

To conclude the proof of Proposition 3, note that by the previous claim and the fact that the domain of ϕ is an interval, Risk Stationarity implies that ϕ is a power function. Moreover, since ϕ is less concave than the log, it can be expressed as $\phi(x) = x^\alpha$ for some $\alpha > 0$. Therefore,

$$\phi(\beta^t u(x)) = \left(\beta^t u(x)\right)^\alpha = \tilde{\beta}^t \tilde{u}(x),$$

where $\tilde{\beta} := \beta^\alpha$ and $\tilde{u}(x) := (u(x))^\alpha$. ■

C.6 Proof of Proposition 4

By Proposition 2, $\succsim$ is RSTL if an only if ϕ is a convex transformation of ln. Recall also that Stochastic Impatience holds if and only if $U(x,t) = \phi(\beta^t u(x))$ has decreasing differences (see equation 4). We now show that $\succsim$ displays Stochastic Impatience if and only if ϕ is a convex transformation of ln.

Let $f : X \times T \to \mathbb{R}$ be given by $f(x,t) \equiv g\left(\ln\left(\beta^t u\left(x\right)\right)\right) = g\left(t \ln\left(\beta\right) + \ln u\left(x\right)\right)$ for some increasing function g. The function f has decreasing differences if and only if

$$f(x_1,t_1) + f(x_2,t_2) \geq f(x_1,t_2) + f(x_2,t_1)$$

for all $x_1, x_2 \in X$ and $t_1, t_2 \in T$ with $x_1 > x_2$ and $t_1 < t_2$. We now verify that f has decreasing differences if and only if g is convex. To see this, rewrite the condition as

$$g(z_{11}) + g(z_{22}) \geq g(z_{12}) + g(z_{21})$$

where $z_{ij} \equiv t_i \ln\left(\beta\right) + \ln u\left(x_j\right)$. Note also that

$$z_{11} \geq \max\left\{z_{12}, z_{21}\right\} \geq \min\left\{z_{12}, z_{21}\right\} \geq z_{22},$$

and

$$z_{11} + z_{22} = z_{12} + z_{21}.$$

By the richness condition, the image of $\psi(x,t) \equiv t \ln\left(\beta\right) + \ln u\left(x\right)$ in $X \times T$ is an interval, so the condition becomes

$$g(z_{11}) + g(z_{22}) \geq g(z_{12}) + g(z_{21})$$

for all $z_{11}, z_{12}, z_{21}, z_{22} \in \psi(X,T)$ such that $z_{11} \geq z_{12} \geq z_{12} \geq z_{22}$ with $z_{11} + z_{22} = z_{12} + z_{21}$, which is true if and only if g is convex. ■

C.7 Proof of Theorems 3 and 4

Recall that, for each fixed background consumption c, we denote by

$$U([c,(x,t)]) \equiv U\left(c, c, ..., \underbrace{c+x}_{t}, c, ..., c\right).$$

the utility of receiving a prize x at date t. The proofs of Theorems 3 and 4 follow the exact same steps as the proofs of Theorems 1 and 2, after replacing $U(x,t)$ by $U([c,(x,t)])$ and holding the background consumption c fixed. ■

D Experiments

D.1 Lab Experiment: additional information

A total of 196 subjects took part in an experiment run at the Wharton Behavioral Lab at the Wharton School of the University of Pennsylvania. We used a paper-and-pencil questionnaire. Some questions involved immediate payments, that were made at the end of each session. Others involved payments to be made in the future; for these, subjects were told that their payment would be available to pick up from the lab starting from the date indicated.[32]

We ran two treatments: 'long delay' and 'short delay,' labeled Long and Short in what follows. A total of 105 and 91 subjects participated in each, respectively. The only difference was the length of delays in some of the questions: in the Long treatment, some payments were delayed by up to 12 weeks, while in the Short treatment the maximum delay was 5 weeks.[33]

The experiment has three parts.[34] Part I asks subjects to choose between different time lotteries and it is the main part of our experiment. For example, the first question asked them to choose between \$15 in 2 weeks or \$15 in 1 week with probability .75 and in 5 weeks with probability .25. Subjects answered five questions of this kind. Table 2 lists the questions asked in each treatment. All questions offered two options that paid the same prize at different dates, where the distribution of payment dates of one option was a mean preserving spread of that of the other. In three questions, one of the options had a known date; in the others, both options had random payment dates. All subjects received the same first question (Question 1 in Table 2) on a separate sheet of paper. The answer to this question is a key indication of the subjects'

[32]All payment dates were expressed in weeks, with the goal of reducing heterogeneity in transaction costs between the dates, under the assumption that students have a regular schedule each week during the semester. An email was then sent to remind them of the approaching date (they were told they would receive it). Subjects were also given the contact details of one of the authors, at the time a full-time faculty at Wharton. Returning to the lab to collect the payment involve transaction costs, a typical concern. However, in our experiment *all* payments related to time lotteries were designed to take place in future dates, thus holding constant the transaction cost. We have already mentioned that a second concern may relate to the use of monetary prizes to study time preferences (Augenblick et al., 2015). Since are interested in studying the relation between risk aversion over time lotteries and atemporal risk aversion, and the latter is defined for monetary lotteries, we focus our experiment on monetary prizes. As we mentioned, to test the robustness of our results to non-monetary setups, we also conducted a real-effort experiment, described in Appendix D.4.

[33]Testing both treatments allows us to study long times spans, where, as we have discussed in Section 2.1, differences between time lotteries become more pronounced; as well as shorter ones, where students' schedules are more stable, reducing heterogeneous sources of variation. Note that all payments were scheduled to take place during the academic school year (while school was in session); in addition, no payment was scheduled during exam week.

[34]Subjects received general instructions and specific instructions about the first part when they entered the room. Separate instructions were distributed before each of the following parts. The order of parts and of questions was partly randomized, as we discuss below.

Table 2: Questions in Part I

	Long Delay				Short Delay			
Q.	$$	Option 1	*vs.*	Option 2	$$	Option 1	*vs.*	Option 2
1	$20	2 wk		75% 1 wk, 25% 5 wk	$20	2 wk		75% 1 wk, 25% 5 wk
2	$15	3 wk		90% 2 wk, 10% 12 wk	$15	3 wk		50% 1 wk, 50% 5 wk
3	$10	2 wk		50% 1 wk, 50% 3 wk	$10	2 wk		50% 1 wk, 50% 3 wk
4	$20	50% 2 wk, 50% 3 wk		50% 1 wk, 50% 4 wk	$20	50% 2 wk, 50% 3 wk		75% 2 wk, 25% 4 wk
5	$15	50% 2 wk, 50% 5 wk		75% 1 wk, 25% 11 wk	$10	50% 2 wk, 50% 5 wk		75% 3 wk, 25% 5 wk

Notes. Each lottery pays the same prize with different delays (in weeks). Subjects in the long delay treatment chose between 'Option 1' and 'Option 2, Long Delay.' Those in the short delay treatment chose between 'Option 1' and 'Option 2, Short Delay.'

preferences, as it captures their immediate reaction to this choice, uncontaminated by other questions.[35]

Table 3: Questions in Part II

	Long Delay			Short Delay		
Q.	Option 1	*vs.*	Option 2	Option 1	*vs.*	Option 2
6	$10 today		x in 2 wk	$10 today		x in 2 wk
7	$10 in 1 wk		x in 2 wk	$10 in 1 wk		x in 2 wk
8	$10 in 1 wk		x in 5 wk	$10 in 1 wk		x in 3 wk
9	$10 in 1 wk		x in 12 wk	$10 in 1 wk		x in 4 wk
10	$20 in 4 wk		$20, x% in 2wk, (1-x)% in 12wk	$25 in 3 wk		$25, x% in 2wk, (1-x)% in 5wk
11	$25 in 2 wk		$25, x% in 1wk, (1-x)% in 5wk	$25 in 2 wk		$25, x% in 1wk, (1-x)% in 5wk

Notes. Questions 6-9 ask the amount $\$x$ that would make subjects indifferent between each option. Questions 10-11 ask the probability x% that would make subjects indifferent between each option. These amounts were determined using MPL.

Parts II and III use the Multiple Price List (MPL) method of Holt and Laury (2002) to measure time and risk preferences separately.[36] Part II measures standard time preferences as well as attitudes towards time lotteries (Question 10 and 11). Part

[35] One potential concern with offering a list of similar questions is that subjects may 'try' different answers even if they have a mild preference in one direction with some hedging concern in mind (Agranov and Ortoleva, 2017).

[36] In a MPL, each question has a table with two columns and multiple rows. The subject is asked to make a choice in each row. One of the options is always the same, while the other gets better and better as we proceed down the rows. These questions are typically interpreted as follows: if a subject chooses the option on the left for all rows above a point, and the option on the right below that point, then the indifference point should be where the switching takes place. Subjects who understand the procedure should not switch more than once. This is indeed the case for the large majority of answers: 13% of subjects gave a non-monotone answer in at least one of the 12 MPL questions, and

Table 4: Questions in Part III

Q.	Option 1	*vs.*	Option 2
12	\$15		x% of \$20, (1-x)% of \$8
13	50% of \$15, 50% of \$8		x% of \$20, (1-x)% of \$8
14	20% of \$15, 80% of \$8		x% of \$20, (1-x)% of \$8
15	\$20		x% of \$30, (1-x)% of \$5
16	50% of \$20, 50% of \$5		x% of \$30, (1-x)% of \$3
17	10% of \$20, 90% of \$5		x% of \$30, (1-x)% of \$3

Notes. Questions ask the probability x% that would make subjects indifferent between each option, determined using MPL. All payments were scheduled for the day of the experiment.

III measures atemporal risk preferences, with payments taking place immediately at the end of the session. These include questions to measure regular risk aversion, as well as Allais' common-ratio-type questions, that allow us to test and quantify violations of Expected Utility theory. Tables 3 and 4 include the list of questions asked in these two parts.

At the end of the experiment one question was randomly selected from Parts I, II, and III for payment. The randomization of the question selected for payment, as well as the outcome of any lottery, was resolved with dice.[37] Crucially, all uncertainty was resolved at the end of the experiment, including the one regarding payment dates. The instructions explicitly stated that subjects would know all payment dates before leaving the room.

The order of parts and of questions within parts was partly randomized at a session level. Because Part I is the key one, all subjects saw it first to avoid contamination. For the same reason, within Part I, Question 1 was always the same. All other elements were randomized.[38] We find no significant effects of ordering.[39]

only 4.6% gave non-monotone answers in more than one. These are substantially lower numbers (i.e., fewer violations) than what previous studies have found (Holt and Laury, 2002). The non-monotone behavior did not concentrate in any specific question. Following the typical approach, these answers are disregarded. Alternatively, we could have dropped any subject that exhibits a non-monotone behavior at least once; this leave our results essentially unchanged.

[37] Specifically, one participant was selected as 'the assistant,' using the roll of a die. This subject was then in charge of rolling the die and checking the outcomes. This was done to reduce the fear that the experimenter could manipulate the outcome. All was clearly explained beforehand.

[38] Specifically: for questions in Part I other than the first, half of the subjects answered questions in one specific order (the one used above), while the other half used a randomized order. In each of them, which option appears on the left and which on the right was also determined randomly. The order of Parts II and III was randomized. For both parts, it was determined randomly whether in the MPL the constant option would appear on the left or on the right. This was done (independently) for each part, but not for each question within a part: in Part II or III the constant option of the MPL was either on the left or on the right for all questions of that part. This is typical for experiments that use the MPL method, as it makes the procedure easier to explain.

[39] The only exception is that out of the five questions in the first part, subjects have a significant

We conclude by noting that our incentive scheme, the random payment mechanism, as well as the Multiple Price List method, are incentive compatible for Expected Utility maximizers, but not necessarily for more general preferences over risk.[40] Since this is the procedure used by most studies, a significant methodological work has been done to examine whether this creates relevant differences, with some reassuring results.[41]

D.2 Lab Experiments: Results

We start from the main variable of interest: risk attitude towards time lotteries. This can be measured in three different ways. First, we can measure it using Question 1 of Part I, the first question that subjects see. Second, we can look at the answers to all five questions in Part I and ask whether subjects exhibited RATL in the majority of them (for the purpose of this section, we say that subjects are RATL in a given question if, in that question, they chose the option with the smallest variance of the payment date). A third way is to look at the answers given in Questions 10 and 11 of Part II, that compute RATL using MPL.

Table 5 presents the percentage of RATL answers for each of these measures. The results are consistent: in most questions, especially in the Long treatment, the majority of subjects are RATL. Note that most subjects are still RATL when *both* options are risky but one of the options is a mean preserving spread of the other (Questions 4 and 5). Thus, the data suggest an aversion to mean preserving spreads, not simply an attraction towards certainty.

In most questions, RATL is stronger in the Long rather than in the Short treatment. We have already discussed in Section 2.1 how this is the opposite of what is predicted by EDU.

While most answers are consistent with RATL, it could be that a non-trivial fraction of our subjects still consistently chooses the risky option, as predicted by EDU. Table 6 shows that this is not the case: the fraction of subjects who does so is

(moderate) preference for the option on the right in the second question. While this is most likely a spurious significance (due to the large number of tests run), the order was randomized for all sessions and thus this should have no impact on our analysis.

[40] Holt (1986) points out that a subject who obeys the Reduction of compound lotteries but violates the Independence axiom may make different choices under a randomly incentivized elicitation procedure than he would make in each choice in isolation. Conversely, if the decision maker treats compound lotteries by first assessing the certainty equivalents of all first stage lotteries and then plugging these numbers into a second stage lottery (as in Segal, 1990), then this procedure is incentive compatible. Karni and Safra (1987) prove the non-existence of an incentive compatible mechanism for general non-Expected Utility preferences.

[41] Beattie and Loomes (1997), Cubitt et al. (1998) and Hey and Lee (2005) all compare the behavior of subjects in randomly incentivized treatments to those that answer just one choice, and find little difference. Also encouragingly, Kurata et al. (2009) compare the behavior of subjects that do and do not violate Expected Utility in the Becker-DeGroot-Marschak procedure (which is strategically equivalent to MPL) and find no difference. On the other hand, Freeman et al. (2019) find that subjects tend to choose the riskier lottery more often in choices from lists than in pairwise choices.

Table 5: Percentage of RATL in each question

Question	Long	Short
1	65.71	56.04
2	50.48	54.95
3	48.57	37.36
4	64.76	38.46
5	73.33	52.75
Majority in 1-5	64.76	49.45
10	44.23	54.44
11	57.28	41.11
Either in 10 or 11	64.07	66.66

Table 6: Frequency of RATL answers in Part I

Frequency of RATL	Long Delay		Short Delay	
	Percent	Cum.	Percent	Cum.
0	2.86	2.86	9.89	9.89
1	9.52	12.38	16.48	26.37
2	22.86	35.24	24.28	50.55
3	23.81	59.05	26.37	76.92
4	28.57	87.62	19.78	96.70
5	12.38	100.00	3.30	100

minuscule in the Long treatment (2.86%) and very small in the Short one (9.89 %). By contrast, in the Long treatment almost 41% give risk averse answers at least 4 out of 5 times, and 59% at least three times. (These numbers are about 23% and 48.45% in the Short treatment.)

Overall, these finding are not compatible with RSTL and thus with EDU: only a minuscule fraction of subjects is consistently risk seeking over time lotteries, while the majority is tends to be risk *averse* over time lotteries. Thus, the assumption of risk seeking over time lotteries, implicitly present when using EDU, does not seem to have a positive appeal. Finally, recall that in Section 2.1 we have already discussed how our data is also not compatible with standard stochastic choice extensions of EDU.

D.3 RATL, Convexity, Expected Utility, and Risk Aversion

We now turn to analyze the relationship between RATL and convex discounting, violations of Expected Utility, and atemporal risk aversion. Under EDU, all subjects with convex discounting should be RSTL; in turn, this means that such tendency should be negatively related to convexity of the discount function. Under GEDU, RATL should be positively correlated with atemporal risk aversion. Finally, if RATL were due to violations of Expected Utility, as suggested by Chesson and Viscusi (2003) and Onay and Öncüler (2007), then it should be linked to certainty bias and violations of Expected Utility.

We quantify convexity of the discount function, violations of Expected Utility, and atemporal risk aversion using the MPL measures collected in Parts II and III. We determine which subjects have convex discounting based on their answers in Part II (see Questions 7, 8, and 9 in Table 3). Unsurprisingly, we find that 82% of our subjects exhibit it (this is an established finding). From the questions in Part III we can construct two related measures of violations of Expected Utility. First,

Table 7: Proportion of RATL subjects

Sample	Convex Discounting		Approximately Exp. Ut.		No Certainty Bias	
Treatment	Long	Short	Long	Short	Long	Short
Question 1	67.78	50.70*	66.67	60.61	67.50	55.00
Majority in Q1-5	65.56	43.66**	64.29	60.61*	68.75*	50.00
Question 10	46.07	52.86	54.76*	68.75**	47.50	51.90
Question 11	57.95	44.29**	64.29	56.25	54.43	48.10
Observations	90	71	42	33	80	80

Notes. The first row measures RATL using Question 1. The second row identifies as RATL subjects who chose the safe option in the majority of Questions 1-5. The third and fourth rows use answers to MPL Questions 10 and 11. Columns present the proportion of RATL subjects in the subsamples of subjects with convex discounting, approximately Expected Utility, and those with no Certainty Bias as measured using Questions 12 and 13. * and ** denote significance at the 10% and 5% level in a Chi-squared test of whether each subset is different from its complement (within the Long or Short treatments).

we can determine if subjects exhibit certainty bias (Kahneman and Tversky, 1979), which is implied by pessimistic probability weighting.[42] We find that a small number of subjects exhibit it (15.71%).[43] Second, we can use the same three questions to determine whether the subjects give answers that are jointly consistent with Expected Utility. Since this is a very demanding requirement—it is well-known that these measures are very noisy—, we consider as "approximately Expected Utility" those subjects who would abide by Independence across all three questions if we changed their answer in at most one of the lines. This holds for 39.89% of the pool.

Table 7 shows that, based on the four different measures, subjects are still RATL in each of the subsamples above. The table also shows the significance of Chi-squared tests on whether subjects consistent with a given property (e.g., Convex Discounting) are statistically different from same-treatment subjects who are not consistent with

[42] This could be done using Questions 12 and 13, or 12 and 14 (see Table 4). Suppose that in Question 12 the subject switches at x_{12}, while in Question 13 she switches at x_{13}. If the subject follows Expected Utility, we should have $2x_{13} = x_{12}$. A certainty-biased subject would instead have $x_{12} > 2x_{13}$; because she is attracted by the certainty of Option 1 in Question 12, she demands a high probability of receiving the high prize in Option 2 to be indifferent. Thus, the answers to Question 12 and 13 allow us to identify subjects who are certainty biased and to quantify it. In what follows, when we need to identify subjects who are certainty biased, we use this measure. A similar measure can be obtained from the answers to Questions 12 and 14; the results are essentially identical. When we need to quantify certainty bias (in the regression analysis), we use instead the principal component of the two measures, which should reduce the observation error (essentially identical results hold using either of the two measures or their average).

[43] These small numbers are not surprising: it is a stylized fact that certainty bias is less frequent when stakes are small, as in this part of our experiment (Conlisk, 1989; Camerer, 1989; Burke et al., 1996; Fan, 2002; Huck and Müller, 2012). See the discussion in Cerreia-Vioglio et al. (2015).

that property. We find a majority of RATL among subjects who either exhibit convex discounting, or who exhibit no certainty bias, or who are "approximately Expected Utility." In most cases there is no significant difference in the proportions of RATL between these groups and their complement. These results are in direct contrast with the predictions of EDU, and with the explanation of RATL suggested by Chesson and Viscusi (2003) and Onay and Öncüler (2007) based on probability weighting: according to the former, there should be no RATL with convex discounting; according to the latter, there should be no RATL without certainty bias, or for subjects that (approximately) follow Expected Utility. Tables 8 and 9 present regression analyses to confirm these results: they show how certainty bias or convexity of discounting is generally not related, or poorly related, to the tendency to exhibit RATL.

Table 8: Probit Regressions: RATL and Convexity and Certainty Bias

Dep. Variable	RATL Q1		RATL Majority Q1-5		RATL Q10		RATL Q11	
Treatment	Long	Short	Long	Short	Long	Short	Long	Short
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Certainty Bias	-.25*	.18	-.20	.18	-.03	.44***	.16	.23
	(-1.94)	(1.17)	(-1.60)	(1.20)	(-.25)	(2.77)	(1.42)	(1.52)
Convexity	4.27*	-4.45	.06	-11.10***	3.47	-1.77	.99	-7.73**
	(1.82)	(-1.29)	(.03)	(-2.86)	(1.64)	(-1.29)	(.46)	(-2.16)
Constant	.19	.28*	.39**	.19	-.26	.16	.19	.12
	(1.07)	(1.82)	(2.15)	(1.20)	(-1.52)	(1.03)	(1.08)	(.79)
Pseudo-R^2	.06	.02	.02	.01	.02	.07	.02	.06
Obs.	92	86	92	86	92	85	92	85

Notes. Dependent variables are indicated in the first row. Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

All our findings thus far are compatible with the GEDU model. However, as we pointed out, GEDU makes one additional prediction: RATL should be related to standard atemporal risk aversion. Tables 10 and 11 present the coefficients from Probit regressions with our four RATL measures as dependent variables and the degree of risk aversion (as measured in Question 12) as the independent variable. Consistently with the model, the coefficients are positive and, with the exception of the Short treatment in Question 1, they are all statistically significant at the 5% level. (Similar results hold constructing risk aversion from other questions, e.g., Question 15, or using a linear probability model.)

To summarize, we find that subjects who are either (i) approximately Expected Utility maximizers, or (ii) satisfy either convex discounting, or (iii) satisfy no certainty bias, also have a tendency to be RATL. In fact, the proportions in these groups are almost identical to the one in the overall population. Regression analysis shows

Table 9: Probit Regressions: RATL and Convexity and Certainty Bias

Dep. Var.	RATL Q.10				RATL Q.11			
Treatment	Long		Short		Long		Short	
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Convexity	3.63*		-1.59		1.13		-7.59**	
	(1.73)		(-0.49)		(0.55)		(-2.20)	
Cert. Bias		-.05		.45***		.19*		.18
		(-0.52)		(2.84)		(1.71)		(1.23)
Constant	-.29*	-.11	.13	.13	.12	.24*	.12	.01
	(-1.79)	(-.88)	(.88)	(.92)	(0.75)	(1.80)	(.81)	(0.05)
Pseudo-R^2	.02	.01	.01	.07	.01	.02	.04	.01
Obs.	101	95	87	87	100	95	87	87

Notes. Same as Table 8. Each regression excludes one dependent variable.

that RATL is unrelated to violations of Expected Utility and generally unrelated to convexity. It is, however, related to (atemporal) risk aversion. These findings are not compatible with RSTL, EDU, or to explanations based on probability weighting, but they are compatible with GEDU.

D.4 Real-Effort Experiment

Our main experiment presented above uses monetary incentives. A recent literature has expressed concerns about the use of such incentives to study intertemporal preferences (Augenblick et al., 2015). To test the robustness of our main finding, we also conducted a real-effort experiment with time lotteries over effort relief. We adapted the design of DellaVigna and Pope (2018) to the study of time lotteries.

Design. We recruited 156 subjects on Amazon Mechanical Turk, focusing on subjects in the United States with at least 100 HITs completed with a 95% success rate. The experiment involved real-effort tasks to be completed in three different periods: one, two, and three weeks after the initial experiment. The real-effort task, adapted from DellaVigna and Pope (2018), each week consisted of alternatively pressing the "a" and "b" buttons on their keyboard 750 times. This task takes about five minutes on average, and participants were informed about both the task and the average time prior to answering any questions..

During the initial experiment, subjects were showed the work schedule, the incentives, and were asked three incentivized questions about preferences over time lotteries, taking the following form. In the first question, they were asked if they preferred to skip work in week 2, or if they preferred to skip work in week 1 or 3 with equal probability. If they chose the latter, the computer will randomize and

Table 10: Probit Regressions: RATL and Atemporal Risk Aversion

Dep. Variable	RATL Q.1		RATL Majority Q.1-5		RATL Q.10		RATL Q.11	
Treatment	Long	Short	Long	Short	Long	Short	Long	Short
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Risk Aversion,	.336**	.175	.308**	.341**	.459***	.435***	.571***	.239*
Atemporal	(2.41)	(1.30)	(2.27)	(2.40)	(3.27)	(2.91)	(3.81)	(1.75)
Constant	.07	-.01	.07	-.34*	-.60***	-.28	-.37***	-.19
	(0.38)	(-0.07)	(0.38)	(-1.79)	(-3.01)	(-1.47)	(-1.87)	(-1.08)
Pseudo-R^2	0.047	0.014	0.040	0.049	0.083	0.076	0.121	0.025
Observations	101	90	101	90	101	89	100	89

Notes. Dependent variables are indicated in the first row. Atemporal risk aversion measure is obtained from Question 12. RATL measures were obtained from Question 1 (Regressions 1 and 2), having chosen the safe option in the majority of Questions 1-5 (Regressions 3 and 4), and MPL Questions 10 and 11 (Regressions 5-8). Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

Table 11: Probit Regressions: RATL and Atemporal Risk Aversion

Dep. Var.	RATL Q.1				RATL Majority Q.1-5			
Treatment	Long		Short		Long		Short	
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cert. Bias		-.19		.18		-.21*		.12
		(-1.56)		(1.20)		(-1.71)		(0.82)
Convexity	3.73*		-4.17		-.39		-10.60***	
	(1.68)		(-1.25)		(-0.19)		(-2.83)	
Constant	.22	.40***	.25*	.19	.41**	.37***	.15	.01
	(1.35)	(2.93)	(1.67)	(1.41)	(2.50)	(2.75)	(1.02)	(.09)
Pseudo-R^2	.02	.02	.01	.01	.01	.03	.07	.01
Obs.	101	95	88	88	101	95	88	88

Notes. Same as Table 8. Each regression excludes one dependent variable.

communicate it to them at the end of the section (a few minutes later). Skipping work meant that subjects were not required to complete the real-effort task but still receive the payment as well as the final bonus (described below). It is easy to see how this constitutes a time lottery, as the benefit (skip work) has a stochastic date. Question 2 offered them the option between skipping work with 50% chance in week 1 and 50% in week 2 vs. 75% in week 1 and 25% in week 3. In question 3 the options were skipping with 66% chance in week 2 and 34% in week 3 vs. 34% in week 1 and 66% in week 3.[44] Subjects were also asked a brief question to test that they are are human and comprehend English. (Everyone passed it.)

At the end of the initial experiment, one of the three questions was randomly selected and subjects were told their work schedule based on the options they chose for that question and the resolution of the lottery. They were paid $5 if they complete the initial survey and $3 per week if they adhered to the work plan, i.e., completed the real-effort task unless they were allowed to skip. Each week, they were also sent a 5 cents reminder of the task to be completed, as well as their entire work schedule. Finally, they received a $5 bonus if they completed the whole plan.

Results. Out of the 156 subjects who participated in the first session, we eliminate 5 as they encountered technical issues, and report the results for the remaining ones. (Results are very similar if we include them.) Of them, 89% (135) of subjects completed at least one week of the task, and 76% (115) the whole work plan. Results are again very similar for either subgroup.

Table 12: Frequency of RATL

Frequency of RATL	Percent	Cumulative
0	21.9%	21.9%
1	27.2%	49.0%
2	33.8%	82.8%
3	17.2%	100%

Table 13: RATL in each question

Question	
1	49.0%
2	51.7%
3	45.7%
Majority of questions	51.0%

Tables 12 and 13 show that in this task as well only a minority of subjects consistently picks the RSTL options, even though this percentage is higher than in the lab experiment (23.8%). Here as well, a majority of subjects chooses the RATL option in the majority of questions (51%). Note also that choosing the RSTL option in one question is positively correlated with choosing it in another question. Indeed, even though the answers to individual questions were not far from 50%, the overall distribution has more mass on the tails (all RATL or all RSTL) than a binomial distribution, for which the frequency of 0 to 3 RATL choices would be (0.125, 0.375,

[44]Note that in Question 3, the second option is a mean-reducing spread of the second one, i.e., it has higher variance but also a marginally lower mean. That is, ties are broken in favor of the RSTL option.

0.375, 0.125). Our observed frequency is statistically different from this distribution (Chi-Square Goodness of Fit, $p = 0.0004$). This suggests that the patterns we observe are not random, but rather are due to one group exhibiting RATL and one group exhibiting RSTL preferences.

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