Altruism, Reciprocal Giving, and Information

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This paper addresses theoretical implications of economic altruism. Specifically, under what conditions can two individuals both strictly prefer to give, rather than receive? With modest assumptions, we find that two individuals cannot both prefer to give to the other. For example, we find that a child will never purchase a gift that the parent could otherwise buy in the marketplace. Using this as a starting point, we consider the three person extension and find that a gift will never pass through the hands of all three individuals, completing a cycle.

In the second half, we concentrate on altruism with imperfect information. With imperfect knowledge regarding preferences, we consider two cases. The first is when a husband assumes his wife has the same preferences as himself, and vice versa. If both have separately additive concave utility functions, we conclude that reciprocal giving equilibria cannot occur. The second case looks at altruistic learning between two individuals and concludes that altruistic individuals want to learn more about “happier-than-average" individuals.

1 Introduction

Connection to previous literature

While many still perceive economic agents to be entirely selfish, economists have a long tradition of researching other-regarding preferences, such as altruism. Starting as early as [Smith, 1976], in which the individual feels empathy toward others, and continuing with influential works by Gary Becker, Kenneth Arrow, Amartya Sen, and Ernst Fehr, altruism has an established history in the economics literature.

The economic literature on altruism is diverse, but the first half of this work focuses mostly on Beckerian altruism. Specifically, in [Becker, 1976] altruism is defined as increased utility when the partner's *consumption* is increased, but in [Becker, 1981] the formal model presented defines

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altruism as in increase in utility when the partner's *utility* increases. Mathematically, these are equivalent to $U_1(c_1, c_2)$ and $U_1(c_1, U_2)$ respectfully.[1]

A close reading suggests that the formal definition of altruism in [Becker, 1981] is later translated into a form equivalent to $U_1(c_1, c_2)$. It is worth noting that in this modified form, there are utility functions in which both people prefer to give to each other. This paper shows that this equilibrium cannot occur with the primitive $U_1(c_1, U_2)$ utility functions, so long as an individual still receives positive (direct) marginal utility from his own consumption. Therefore, a reciprocal giving equilibrium with Beckerian Altruism is not possible. We then goes on to show that a three person Becker altruistic group cannot have a cycle in which a dollar is continuously passed around and never spent.

Altruism, however, is particularly prone to information problems. [Waldfogel, 1993] gives a good summary of how a lack of information about preferences can lead to deadweight loss in gift giving. While this is an interesting problem, we approach imperfect altruism in a slightly different way. Specifically, we look to Smith's *Theory of Moral Sentiments* and his approach on altruism, placing it in a mathematical framework. For the most part, however, imperfect altruism has not been featured in previous economic literature.

Overview of Paper

This paper considers mutually recursive utility functions with a single good and is split into three main sections, followed by concluding remarks. The first assumes perfect information and looks at two person groups. This section rules out reciprocal giving equilibria when goods cannot be shared between individuals and for most cases in which they can. The next section considers three person groups and excludes the possibility of "giving cycles" where goods are passed from one person to the next indefinitely. The third section considers imperfect information and introduces two new sources of altruistic information. First, I paramterize altruism from Smith's *Theory of Moral Sentiments*, under which every individual assumes all other individuals' preferences are identical to his own. Secondly, we consider the case in which the individual assumes another person has the preferences of the "average" person. This is followed by concluding remarks and an appendix consisting of some general definitions and an alternative proof to one of the theorems.

2 Two Person Altruism

Under what conditions can two altruistic individuals, such as a husband and wife, both prefer to give to the other? This can be answered by looking at each person's optimal consumption bundle of (person 1's consumption, person 2's consumption).[2] We plot these two optimizations on the same graph as shown below:[3]

[1] And while in [Becker, 1976] $\frac{\partial U_1}{\partial c_2}$ is the Marginal Utility for increasing c_2, in [Becker, 1981], this value is zero everywhere using the formal definition of partial derivatives. However, [Becker, 1981] uses this partial as part of the equilibrium condition (equation 4), and also in [Becker, 1991] in footnote 6 of Chapter 8. Furthermore, [Becker, 1981] reverts back to the formulation of $U_1(c_1, c_2)$ in equations (9) through (14) on pages 6 to 8.

[2] That is, the optimal bundles giving each person full control of both consumptions.

[3] Also featured in *A Treatise on the Family*, [Becker, 1991].

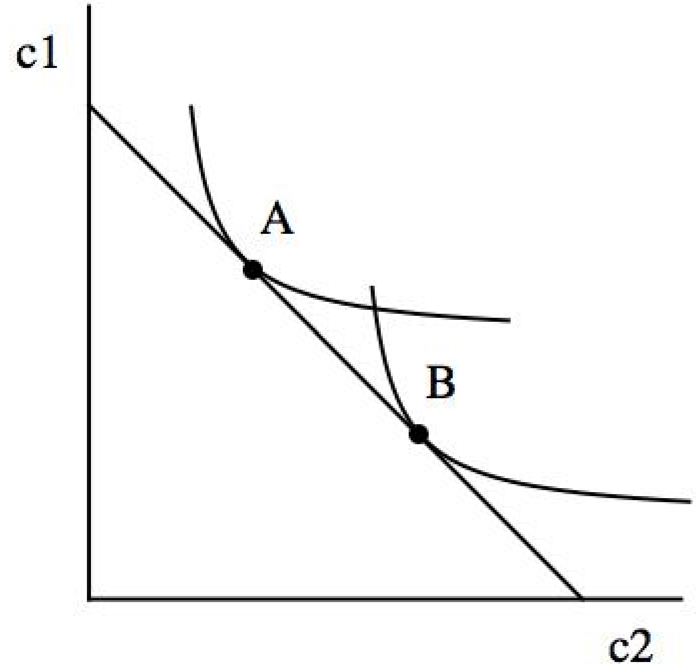

Fig. 1

There are two basic possibilities that follow,[4] each with three possible subcases regarding initial endowments.

(**Case 1**) Let person 1's optimal point be A in the above figure, and person 2's optimal point be B.

(1.a) If the initial endowments are on the budget line to the left of A, then person 1 would prefer that person 2 have more consumption (as person 1's optimal point is A). This results in person 1 preferring to give to person 2. This does not mean, however, that person 2 must accept the gift; person 2 must also prefer to increase his own consumption rather than person 1's consumption. This happens to be true for this subcase: person 2's optimal point has more consumption for himself and less for person 1, so person 2 would prefer to receive gifts from person 1. Looking at person 1 again, we see that if he gives a small amount, he would still be in the region to the left of point A. So he continues to give until he reaches his optimal point. At this point, person 1 is at his optimal point and would no longer prefer to give to person 2. Person 2 would like to receive more gifts but cannot force person 1 to give him more.

(1.b) If the initial endowments are to the right of B, we have a very similar result. Person 2 would like to give to person 1 and person 1 would like gifts from person 2. As a result, person 2 will continue giving until he reaches B. At this point person 2 is at his optimal point and person 1

[4] Not including the trivial case that A and B are the same point.

would prefer more gifts.[5]

To review, these previous two subcases have an equilibrium in which one person prefers more and the other person no longer prefers to give.

(1.c) If the initial endowments are anywhere on the budget line between A and B, neither person would prefer to give, as both individuals would prefer more consumption for themselves. This is also an equilibrium, as it is not optimal for either person to give up consumption.

(**Case 2**) Let person 1's optimal point be B in the above figure and let person 2's optimal point be A.

(2.a) If the initial endowments are to the left of A, person 1 would have most of the initial goods. Person 1 would prefer to give to person 2 and person 2 would prefer to accept (as both individuals' optimal bundle have person 2 consuming more). Therefore person 1 would give to person 2 and we would start sliding down the budget curve. However, once we reach point A, person 1 would prefer person 2 to consume more, but person 2 has already reached his optimal point. That is to say, person 2 would turn down person 1's gifts.

(2.b) If the initial endowments are to the right of B, person 2 would have most of the initial goods. Person 2 would prefer to give to person 1 and person 1 would prefer to accept (again, as both optimal bundles have person 1 consuming more). Therefore person 2 would give to person 1 and we would slide up (left) the budget curve. Upon reaching point B, person 2 would still prefer to give more, but person 1 would not want to deviate from his optimal point, point B.

These previous two equilibria consist of one person preferring to give and the other person having reached his optimal point.
(2.c) If the initial endowments are between A and B, then person 1 would prefer that person 2 have more consumption (as his optimal bundle is to the right of this initial endowment) and person 2 would prefer person 1 have more consumption (as his optimal bundle is to the left of this initial endowment). This is a case in which both individuals prefer the other person to have more consumption and would be an equilibrium as both individuals would be unwilling to accept any gifts from the other person, although both prefer to give gifts to the other.

This section of the paper shows that Case 2 and all of its subcases cannot occur under some basic assumptions and the formal definition of Beckerian Altruism as stated in *A Treatise on the Family*.[6] This is proven very generally, without any assumptions regarding functional form. We then loosen this formal definition of altruism to include direct utility from the other individual's consumption and explore conditions under which Case 2 cannot occur.

[5] As person 1's optimal bundle has more person 1 consumption and less person 2 consumption than person 2's optimal bundle.

[6] [Becker, 1991]

Two Person Case with No Shared Goods

Let us now consider two individuals whose own consumptions do not directly influence the utility of the other individual. However, an increase in the utility of either person increases the utility of the other. Simply put, we will allow for altruism and gift giving, but each individual's consumption is consumed only by himself.

Mathematically, consider two individuals, who consume divisible c_1 and c_2, are altruistic toward one another, and have differentiable and mutually recursive U_1, U_2. With a budget constraint, our model[7] is

$$\begin{aligned} U_1(c_1, U_2(c_2, U_1)) \\ U_2(c_2, U_1(c_1, U_2)) \\ c_1 + c_2 \leq I \end{aligned}$$

We shall also use the following assumptions,[8]

$$\frac{\partial U_1}{\partial U_2} \geq 0, \quad \frac{\partial U_2}{\partial U_1} \geq 0 \quad \forall U_1, U_2 \tag{1}$$

$$\frac{dU_i}{dc_i} \geq 0 \quad i = 1, 2 \tag{2}$$

$$\forall c_1, c_2 \quad \frac{\partial U_1}{\partial c_1} \geq 0 \quad \text{or} \quad \frac{\partial U_2}{\partial c_2} \geq 0 \tag{3}$$

(1) limits this discussion to cases of altruism.

(2) is standard; the marginal utility is greater than zero. If a person has negative marginal utility from consumption, then that person would be better off throwing the dollar away.[9] While an individual might become satiated with respect to one good, I am here lumping all goods into one good.

(3) is a bit harder to justify. In cases where the altruism $\frac{\partial U_1}{\partial U_2}$ is equal to zero, then $\frac{\partial U_1}{\partial c_1} = \frac{dU_1}{dc_1}$. Without altruism, this means that individuals cannot be satiated. If this assumption is then acceptable before considering altruism, it seems odd that altruistic individuals might get sick of spending dollars, holding their partner's utility constant. If this assumption was false, you would have a man who would buy himself CDs in front of his wife, only to return them the next day without her knowledge (i.e. to lead her to believe he had a greater consumption than he actually did). In simple terms, if a person would always like another dollar before getting married, then it seems reasonable that this person would still take another dollar after getting married, if his wife never found out. While there are mathematical forms of altruism for which this does not hold true, I do not believe these accurately represent altruism or human behavior and they are excluded from the results of this theorem.

[7] A one-good Beckerian Altruism model, similar to those in [Becker, 1991].

[8] A proof with slightly different assumptions is presented in the appendix.

[9] One could consider this consumption by considering the good "flushing a dollar down the toilet."

Apparently Rotten Kid Theorem. *With these assumptions, we cannot have an equilibrium in which person 1 strictly prefers to give to person 2 and person 2 strictly prefers to give to person 1. That is, if one person offers, the other person will always accept, or, at worst, be indifferent. In the case of a parent and his child, the child will never give the parent any money or gifts available in the marketplace.*

Proof. Assume for the sake for the sake of contradiction,

$$MU_2(c_1) > MU_2(c_2) \text{ and } MU_1(c_2) > MU_1(c_1)$$

Where $MU_i(c_j)$ is the marginal utility person i gets from increasing person j's consumption an infinitesimal amount. Mathematically, $MU_i(c_j) = \frac{dU_i}{dc_j}$ given that $\frac{dc_m}{dc_n} = 0 \; \forall m \neq n$. That is, a dollar given to another person does not reduce one's own consumption. In other words, marginal utilities do not measure changes in utility along the budget constraint. Instead, it measures the case in which someone's consumption is increased without altering any other individual's consumptions. Using the mathematical description of marginal utilities as provided above,

$$\frac{dU_1}{dc_1} = \frac{\partial U_1}{\partial c_1} + \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_1} \geq \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_1} > \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_2} = \frac{dU_1}{dc_2}$$

Which is a contradiction.[10] This result is achieved regardless of initial endowments, as long as all trades between the two individuals are voluntary. Therefore, since this equilibrium cannot be had for any initial endowment, Case 2 from above can never occur (as it has initial endowments which result in such an equilibrium). □

Let us consider the classic parent and child case. Assuming children have no income of their own, a child will never give the parent any money or gifts available in the marketplace. Therefore, every child will appear rotten in this sense, even if he is extremely altruistic. The last portion of this section, entitled "Gifts", delves further into why this may not seem immediately intuitive.

From page 292 of *A Treatise on the Family*,[11] Becker writes,"The Rotten Kid Theorem does not imply, however, that families with altruistic members are perfectly harmonious. Selfish children want larger contributions from their parents, selfish wives want larger contributions from their husbands,..." However, it is not just selfish children that want larger contributions, but rather all no-income members of a household will always prefer to receive more gifts (i.e. a larger contribution). In simple terms, if an income-earner is willing to give, any no-income member will always accept.

However, the Apparently Rotten Kid (ARK) Theorem does not imply the child would steal from the parent, even if there was no chance of getting caught and an absence of moral costs like guilt. Instead, the parent's offer to give the child a dollar informs the child that the parent would attain higher utility if the child were to consume the dollar (rather than the parent consuming it himself). The child, who would be happier with more consumption regardless of the utility of the parent, is also encouraged to take the dollar to maximize the parent's utility.

[10] If, instead, $\frac{\partial U_1}{\partial c_1} < 0$ and we had $\frac{\partial U_2}{\partial c_2} > 0$, there is a symmetric proof by exchanging all 1's with 2's and all 2's with 1's.

[11] [Becker, 1991]

It should also be mentioned that this theorem does allow for some uncertainty regarding the other individual's utility. That is, the result holds as long as person 1's expectations of U_2 are a function $f(U_2)$ with f differentiable, and $f' > 0$. We explore other cases of imperfect information in the second half of the paper.

Lastly, there is an interesting concept at play here. With the above, it can be derived that

$$\frac{dU_1}{dc_1} = \frac{1}{1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}} \frac{\partial U_1}{\partial c_1}$$
$$\frac{dU_2}{dc_2} = \frac{1}{1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}} \frac{\partial U_2}{\partial c_2}$$

That is to say, the total marginal utility derived from an increase in consumption depends on the direct marginal effect multiplied by a term that is reminiscent of a "multiplier effect". As such, I occasionally refer to $\left(1 - \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}\right)^{-1}$ as the "altruism multiplier" even though the partials are functions. This altruism multiplier effect captures the utility increase due to the reciprocal nature of altruism. This effect can also be captured through a geometric sum, as you have:

$$\begin{aligned}\frac{dU_1}{dc_1} &= \frac{\partial U_1}{\partial c_1} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}\frac{dU_1}{dc_1} \\ &= \frac{\partial U_1}{\partial c_1} + \frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}\frac{\partial U_1}{\partial c_1} + \left(\frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}\right)^2 \frac{dU_1}{dc_1} \\ &= \sum_{i=0}^{\infty} \left(\frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}\right)^i \frac{\partial U_1}{\partial c_1}\end{aligned}$$

The $\frac{\partial U_1}{\partial U_2}\frac{\partial U_2}{\partial U_1}$ is the term which captures the back and forth rise in utility. For example, momentarily equating utility with happiness, the husband is pleased that the wife is pleased due to a rise in the husband's consumption. This is the first degree of altruism. But, then the husband becomes happier because the wife grew happier at the fact that the husband was pleased that the wife is pleased due to a rise in the husband's consumption, a second degree of altruism. This continues on indefinitely, in theory.

Alternatively, consider for a moment we give a candy bar to a friend. We see his smile, which is a good first indicator. We also start to smile at this smile, which causes the friend to smile wider. We then smile wider. The width of our smiles would converge. Suppose instead that the width of a smile is not enough information. So we ask, "Are you enjoying the candy bar?" The friend replies, "Yes, I am. I enjoy it as much as I would four bananas."[12] We are made happy and then ask, "Are you happy that I'm happy that you're happy from eating the candy bar?" The patient friend replies, "Why yes, this also makes me happy." This conversation quickly becomes something you might expect from two lovers, head over heels for each other.

Two Person Case with Shared Consumption

[12] It just so happens our friend is also an economist.

In this case, we allow each individual to also directly enjoy the consumption of the other person.[13] For example, consider c_i to also include durable goods that can be reused, like kitchen appliances, or goods that can be easily shared, such as music CDs, plants, or lighthouses. Given the following assumptions, the potential equilibrium in which each person strictly prefers to give to the other person is not feasible.

Thus, our model addresses utility functions of the following type:

$$\begin{aligned} U_1(c_1, c_2, U_2(c_1, c_2, U_1)) \\ U_2(c_1, c_2, U_1(c_1, c_2, U_2)) \\ c_1 + c_2 \leq I \end{aligned}$$

In addition, we have the following restrictions.

$$\frac{\partial U_1}{\partial U_2} > 0 \quad \frac{\partial U_2}{\partial U_1} > 0 \quad \forall U_1, U_2 \tag{4}$$

$$\frac{dU_i}{dc_i} \geq 0 \quad i = 1,2 \tag{5}$$

$$\frac{\partial U_1}{\partial c_1} \geq 0 \quad \frac{\partial U_2}{\partial c_2} \geq 0 \quad \forall c_1, c_2 \tag{6}$$

$$\forall c_1, c_2, \text{ either} \quad \frac{\partial U_1}{\partial c_1} > \frac{\partial U_1}{\partial c_2} \text{ or } \frac{\partial U_2}{\partial c_2} > \frac{\partial U_2}{\partial c_1} \tag{7}$$

(4),(5), and (6) are similar to the two person case without shared consumption.[14]

(7) implies that there is a logical ordering of the individuals' consumptions. If there are c_1 and c_2 such that (7) is false, then person 2 is directly enjoying person 1's consumption more than his own[15] and vice versa. As c_i represents the vector of all goods purchased, this implies that person 2 should just mimic person 1's consumption choices (that is, the money should go toward what person 1 would have otherwise bought). This assumption might be false if there are barriers that prevent people from purchasing what they would like to. This seems unlikely to be true, particularly for both individuals. Furthermore, on the chance this assumption is false for all c_1, c_2, then they can be relabeled as $c_2' = c_1, c_1' = c_2$, which confers proper 'ownership' of consumption.

Shared Goods Extension to the ARK Theorem. *With the above assumptions, person 1 and person 2 will never both prefer to give to each other. If we consider c_i to be person i's movie rental choices, then we will never have both individuals offering to let the other person pick what to watch, even if both people still enjoy the other person's choice.*

Proof. Assume for contradiction, $\frac{dU_2}{dc_1} > \frac{dU_2}{dc_2}$ and $\frac{dU_1}{dc_2} > \frac{dU_1}{dc_1}$. Without loss of generality, assume the first case of (7) is true. The proof is symmetric if it is the second case of (7) that is true.

$$\Rightarrow \frac{dU_1}{dc_1} = \frac{\partial U_1}{\partial c_1} + \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_1} > \frac{\partial U_1}{\partial c_2} + \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_2} = \frac{dU_1}{dc_2}$$

[13] We still allow an increase in the utility of the partner to increase the utility of the individual.

[14] A subset of the issue at hand.

[15] Keeping person 1's utility constant.

Which is a contradiction. Hence, with the earlier assumptions, we cannot have both $\frac{dU_2}{dc_1} > \frac{dU_2}{dc_2}$ and $\frac{dU_1}{dc_2} > \frac{dU_1}{dc_1}$ be true. $\square$

With the above results, a child will not give a gift with optimizing behavior. That is to say, any no-income member will always stop receiving gifts before he has enough consumption that he starts preferring to give it away. This may not match personal experience, and so this section discusses these incongruities.

One possible reason that a child might give a parent a gift is that the child's gifts cannot be purchased on the market. These gifts extend beyond the deadweight loss of gift-giving as in [Waldfogel, 1993]; such a loss might not occur when the good cannot be purchased by the gift-receiver. The parent might be willing to compensate the child for such gifts through gifts of his own. [16]

3 Three Person Group Altruism

Now let us consider the three person altruism case in which each individual has his own consumption that does not directly influence the utility of the others. As before, an increase in an individual's utility increases the utilities of the others.

That is to say, consider three individuals who consume divisible c_1, c_2, and c_2 respectively, who are altruistic toward one another, and have differentiable and mutually recursive U_1, U_2, U_3 of the forms

$$\begin{aligned} U_1(c_1, U_2(c_2, U_1, U_3), U_3(c_3, U_1, U_2)) \\ U_2(c_2, U_1(c_1, U_2, U_3), U_3(c_3, U_1, U_2)) \\ U_3(c_3, U_1(c_1, U_2, U_3), U_2(c_3, U_1, U_3)) \\ c_1 + c_2 + c_3 \leq I \end{aligned}$$

With the following assumptions, we see that we cannot have a 3 person giving cycle.[17] In layman's terms, a dollar cannot be passed around indefinitely from person to person in a cycle.

[16] Alternatively, consider the gift as a signal that informs the parent about the altruism of the child, or so the parents think. That is, the parent takes gift-giving from the child to the parent as an estimator for the altruism of the child to the parent. If a child gives more gifts, perhaps the altruism of the parent increases, causing the parent to prefer to give more gifts. For example, we might end up with the child refusing the parent's offer to increase the child's consumption today, on the basis that tomorrow the parent might offer three times as much, due to the initial rejection. In this light, gifts are used by children as a technique to get future consumption from the parent. This matches anecdotal observations, as a child's gift is usually income-cheap but time-expensive. Thus, the child spends time to receive a higher income. The child would be unlikely to give a monetary gift, as this is a more costly investment for the child.

[17] A *giving cycle* is formally defined here to be a set $\mathbb{S} \subset \mathbb{Z}^2$ of pairs (i,j) with the following conditions: 1) person i would rather have person j consume a dollar than to spend it on himself if and only if (i,j) $\in \mathbb{S}$ and 2) $\forall$(i,j)$\in \mathbb{S}$, $\exists k, z \in \mathbb{Z}$ such that (k,i) and (j,z) $\in \mathbb{S}$.

We shall also use the following assumptions,

$$\frac{\partial U_i}{\partial U_j} \geq 0 \quad \forall i, j, U_i, U_j \tag{8}$$

$$\frac{dU_i}{dc_i} \geq 0 \quad i \in \{1,2,3\} \tag{9}$$

These assumptions are extensions to the assumptions given in the two person case with no shared consumption.

Nonexistence of 3-person Cycles. *With three people, there is the possibility of person 1 preferring to give to person 2 (rather than consume or give to person 3), person 2 preferring to give to person 3, and person 3 preferring to give to person 1, thus keeping money being passed from hand to hand, trapped in a vicious cycle. However, given the above assumptions, this cannot happen. Furthermore, if person 2 would prefer to give to person 3, neither person 3 nor person 1 can prefer to give to person 2. Put simply, a giver will not be given any gifts.*

Proof. Assume, for the sake of contradiction:

$\frac{dU_1}{dc_2} > \max(\frac{dU_1}{dc_1}, \frac{dU_1}{dc_3})$ and $\frac{dU_2}{dc_3} > \max(\frac{dU_2}{dc_1}, \frac{dU_2}{dc_2})$

This implies $\frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_2} + \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_2} = \frac{dU_1}{dc_2} > \frac{dU_1}{dc_3} = \frac{\partial U_1}{\partial U_2}\frac{dU_2}{dc_3} + \frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_3}$

And, from above, $\frac{dU_2}{dc_3} > \frac{dU_2}{dc_2}$,

$\Rightarrow \frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_1} > \frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_3}$

$\Rightarrow \frac{dU_3}{dc_1} > \frac{dU_3}{dc_3}$

However, as

$\frac{\partial U_2}{\partial U_1}\frac{dU_1}{dc_3} + \frac{\partial U_2}{\partial U_3}\frac{dU_3}{dc_3} = \frac{dU_2}{dc_3} > \frac{dU_2}{dc_1} = \frac{\partial U_2}{\partial U_1}\frac{dU_1}{dc_1} + \frac{\partial U_2}{\partial U_3}\frac{dU_3}{dc_1}$

And $\frac{dU_1}{dc_2} > \frac{dU_1}{dc_1}$

$\Rightarrow \frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_3} > \frac{\partial U_1}{\partial U_3}\frac{dU_3}{dc_1}$

$\Rightarrow \frac{dU_3}{dc_3} > \frac{dU_3}{dc_1}$

Which contradicts the earlier result. Therefore, we cannot have $\frac{dU_1}{dc_2} > \max(\frac{dU_1}{dc_1}, \frac{dU_1}{dc_3})$ and $\frac{dU_2}{dc_3} > \max(\frac{dU_2}{dc_1}, \frac{dU_2}{dc_2})$. In words, we cannot have person 1 strictly preferring to give to person 2 and person 2 strictly preferring to give to person 3. From this the full cycle, which also includes person 3 strictly preferring to give to person 1, cannot occur. □

The three person case is a bit different from the two person case in that a dollar could potentially be trapped, cycling from person to person without being spent. However, as we have seen, these cycles are not possible with altruism, at least for 3 people with optimizing behavior. I believe the economic intuition of this result is as follows: If person 1 would rather give money to person 2, then person 1 weighs an increase in person 2's utility heavily. However, if person 2 would rather give money to person 3 than spend it on himself, then person 1 would make person 2 best off by giving person 3 the money.

For example, consider a husband, a wife, and their mutual friend. The wife comes into her inheritance and, in the absence of the mutual friend, wants to buy her husband a new car. Her husband, however, secretly loves the mutual friend and would rather buy the mutual friend the car. The wife will buy the car for the mutual friend rather than her husband, as she primarily wants to make the husband happy. This may not seem intuitive, but consider that if a husband cares that much for the third party, it might suggest he does not care as much about his wife. Altruism is hardly immutable; this information may come as a shock to the wife and her altruism toward the husband might be greatly reduced. However, if the wife's love was "unconditional", in the sense that she truly just wants to make her husband happy and her love is impervious to new information, she would buy the mutual friend the car. Some examples of imperfect altruism, which seem much more fitting, are shown in the second half of the paper.

Also note that in the real world, it is possible that person 1 would give person 2 the money, knowing full well that person 2 will then give it to person 3. However, if there was some transfer cost, such as postage to send the package, then person 1 would give the gift directly to person 3, assuming person 2 has perfect information about person 3's consumption levels. Without transfer costs, a good might be passed through many hands, but it will never complete a cycle.

This also holds for N-person chains, but the assumptions on the altruism are slightly different. I plan to complete this line of inquiry in a later work.

4 Imperfect Altruism

This section extends previous sections by allowing for imperfect information about other individual's utility functions.[18] For example, a husband may never fully know the utility function of his wife, and instead be forced to rely on expectations. The notation is as follows:

$$U_1(c_1, E_1[U_2])$$
$$U_2(c_2, E_2[U_1])$$

As mentioned in a previous section, if $E_i[U_j] = f_i(U_j)$ with f_i differentiable and increasing, the previous results hold true. However, this section considers other interesting possibilities, as follows: (I) Individuals use their own preferences to estimate the utility of their partner. (II) Individuals use, in part, the preferences of the "average person" (or U_{norm}) to estimate their partner's utility. (III) Individuals receive less utility from their partner as the spatial distance between them increases.

This paper does not consider imperfect knowledge of c_i, although this is almost certainly the case for the majority of altruistic relationships. I hope to examine possible results in a later work.

[18] For the sake of simplicity, I stick to the 2 person case from here on out.

Smithian Altruism

In Smith's *Theory of Moral Sentiments*, an individual does not care about the actual happiness of the other person. Instead, he evaluates his own empathetic utility function at the level of consumption of the other person. Smith says it best, in [Smith, 1976] from [I.i.1.10-11],

> Sympathy, therefore, does not arise so much from the view of the passion, as from that of the situation which excites it. We sometimes feel for another, a passion of which he himself seems to be altogether incapable; because, when we put ourselves in his case, that passion arises in our breast from the imagination, though it does not in his from the reality. We blush for the impudence and rudeness of another, though he himself appears to have no sense of the impropriety of his own behaviour; because we cannot help feeling with what confusion we ourselves should be covered, had we behaved in so absurd a manner. [...] The compassion of the spectator must arise altogether from the consideration of what he himself would feel if he was reduced to the same unhappy situation, and, what perhaps is impossible, was at the same time able to regard it with his present reason and judgement.

This could also be considered as empathetic altruism or Kantian altruism. With this in mind, we have

$$\begin{gathered} U_1(c_1, U_1(c_2, U_1)) \\ U_2(c_2, U_2(c_1, U_2)) \\ c_1 + c_2 \leq I \end{gathered}$$

This means that each person assumes the other person has preferences identical to his own. In other words, person 1 expects person 2's utility to be what person 1's utility would be if person 1 was consuming c_2. Person 1 also assumes that person 2 has the same level of altruism that person 1 would have if he were in person 2's situation. In math, $E_1[U_2] = U_1$

Notation: For this section, I will let $U_1(c_1, U_1)$ be denoted as U_1, and $U_1(c_2, U_1)$ as $\widetilde{U_1}$. Therefore, $U_1 = U_1(c_1, \widetilde{U_1}) = U_1(c_1, U_1(c_2, U_1(c_1, \widetilde{U_1})))$

Consider the case in which

$$U_1(c_1, \widetilde{U_1}) = f(c_1) + g(\widetilde{U_1})$$

I.e. U_1 is additively separable. Furthermore, consider $f' \geq 0$, $f'' \leq 0$, for all c_1, and $g' \geq 0$, $g'' \leq 0$ for all U_1. Lastly, we must assume $\frac{dg}{dU_1}\frac{dg}{d\widetilde{U_1}} \neq 1 \quad \forall c_1, c_2$. If this was not the case, the recursive nature causes the utility to be undefined. Assume similar conditions for individual 2 (additively separable concave functions).

Smithian Limit Theorem. *With additively separable concave, differentiable, and increasing functions, two individuals with Smithian preferences cannot both prefer to give to one another. Furthermore, with these preferences, an individual can only prefer to give to his partner if his partner's consumption is less than his own.*

[18] Probably not important enough to give it such a weighty name.

Proof. It might be useful to recall that, unlike total derivatives, $\frac{\partial U_1}{\partial U_1}$ is not necessarily 1 and that we are moving on with $\frac{dc_m}{dc_n} = 0\ \forall m \neq n$, i.e. marginal utilities.

$$\frac{dU_1}{dc_1} = \frac{df(c_1)}{dc_1} + \frac{dg}{d\widetilde{U_1}}\frac{d\widetilde{U_1}}{dc_1}$$

$$\frac{d\widetilde{U_1}}{dc_2} = \frac{df(c_2)}{dc_2} + \frac{dg}{dU_1}\frac{dU_1}{dc_2}$$

$$\frac{dU_1}{dc_2} = \frac{dg}{d\widetilde{U_1}}\frac{d\widetilde{U_1}}{dc_2}$$

$$\frac{d\widetilde{U_1}}{dc_1} = \frac{dg}{dU_1}\frac{dU_1}{dc_1}$$

These give us:

$$\frac{dU_1}{dc_1} = \frac{1}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_1)}{dc_1}$$
$$\frac{d\widetilde{U_1}}{dc_2} = \frac{1}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_2)}{dc_2}$$
$$\frac{dU_1}{dc_2} = \frac{\frac{dg}{d\widetilde{U_1}}}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_2)}{dc_2}$$
$$\frac{d\widetilde{U_1}}{dc_1} = \frac{\frac{dg}{dU_1}}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_1)}{dc_1}$$

Given that $\frac{dU_1}{dc_1} \geq 0$ (i.e. marginal utility to consume is greater than or equal to 0) and that $\frac{df(c_1)}{dc_1} \geq 0$ (from earlier assumptions about concavity), it follows that $\frac{1}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}} \geq 0$ which implies $\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1} < 1$. We shall come back to this result shortly.

So, if person 1 would prefer to give to person 2, we have:

$$\frac{dU_1}{dc_2} > \frac{dU_1}{dc_1} > 0$$
$$\frac{dU_1}{dc_2} = \frac{\frac{dg}{d\widetilde{U_1}}}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_2)}{dc_2} > \frac{1}{1-\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1}}\frac{df(c_1)}{dc_1} = \frac{dU_1}{dc_1} > 0$$
$$\frac{dg}{d\widetilde{U_1}}\frac{df(c_2)}{dc_2} > \frac{df(c_1)}{dc_1} > 0$$

This results in two possibilities. Consider case 1, where $\frac{dg}{d\widetilde{U_1}} \leq 1$. Then,

$$\frac{df(c_2)}{dc_2} \geq \frac{dg}{d\widetilde{U_1}}\frac{df(c_2)}{dc_2} > \frac{df(c_1)}{dc_1} > 0$$
$$\Rightarrow c_2 < c_1, \text{ as f concave and increasing.}$$

Therefore, in case 1, if we have person 1 preferring to give to person 2, person 1 must have a larger consumption. Now consider case 2, where $\frac{dg}{d\widetilde{U_1}} > 1$. As stated earlier, we have $\frac{dg}{d\widetilde{U_1}}\frac{dg}{dU_1} < 1$, so this means

$$\frac{dg}{d\widetilde{U_1}} > 1 > \frac{dg}{dU_1}$$
$$\Rightarrow g(\widetilde{U_1}) < g(U_1(c_1, \widetilde{U_1})) \text{ as g concave.}$$
$$\widetilde{U_1} < U_1(c_1, \widetilde{U_1}) \text{ as g increasing.}$$
$$f(c_2) + g(U_1(c_1, \widetilde{U_1})) < f(c_1) + g(\widetilde{U_1})$$

But just three lines above we saw that $g(\widetilde{U_1}) < g(U_1(c_1, \widetilde{U_1}))$, which implies (in conjunction with the previous line):

$$f(c_2) + g(U_1(c_1, \widetilde{U_1})) < f(c_1) + g(U_1(c_1, \widetilde{U_1}))$$
$$\Rightarrow f(c_2) < f(c_1)$$
$$\Rightarrow c_2 < c_1$$

Therefore, in both cases, we end up with the requirement that person 2's consumption is less than person 1's. It follows that if person 2's consumption is greater than person 1's, then person 1 will never prefer to give to person 2 (given the additively separable concave functions).

This result is symmetric, given that person 2 also has additively separable concave functions. That is to say, in order for both people to prefer to give to the other person, both individuals must have greater consumption than the other, an obvious contradiction. □

Smithian altruism can also be considered misguided altruism. The issue is that person 1 and person 2 lack[19] proper information about what the other person actually wants. For example, a nonsmoker may see a friend smoking and, substituting her own preferences for what her friend desires, ask her friend to quit. However, if the friend did have the same preferences, she would have quit already. This suggests the friend does not lack information about what the other person prefers, but instead forces her preferences upon her friend. That being said, this is a complex case as there are other possible explanations to why a nonsmoker would ask a smoker to quit. [20]

Without the assumptions of concavity, we do not necessarily have this result. Consider $U_1(c_1, \widetilde{U_1}) = \frac{1}{2}c_1^2 + \frac{1}{2}\widetilde{U_1}$. Now, the marginal utility for c_1 is $\frac{4}{3}c_1$ and the marginal utility for c_2 is $\frac{2}{3}c_2$. Therefore, if $c_2 > 2c_1$ we have a case in which person 1 would prefer to give to person 2, even though person 2 has more consumption.

Benefits of Altruistic Information

To briefly review, we have two individuals who are altruistic in the sense that when they believe their partner has a higher utility, they themselves have a higher utility. These expectations could be based on anything, but I propose three different ways of estimating this expectation. The first is when one individual knows the other very well and can base his expectation upon the actual U_i, as was the case in the first half of the paper. The second is when one individual imposes his own

[19] Or disregard.

[20] Primarily, there might be a misunderstanding about the consumption of the smoker. For example, the non-smoker does not know that the smoker has a lot of health insurance, or that the smoker grew up in a household of smokers. Of course, second hand smoke may also be an issue.

preferences upon the other, which I have called Smithian Altruism above. The third is when one individual bases his expectations upon what he considers the “normal" person. That is to say, if one believes himself to have peculiar tastes, he may not impose his own preferences upon another, assuming instead that his partner has preferences close to the average person. Therefore, he may base his expectation upon U_{norm}.

I believe all three are used at various times or by different people; a father may impose his own preferences on his child, even if the father knows his own preferences lie far from the norm. On the other hand, buying a gift for a six year old niece may involve using one's expectations of what a normal six year old girl would like.

Let us now look at this as an information problem, using the general definitions found at the start of the section. Assume that $E_1[U_2] = \alpha_1 U_2 + (1-\alpha_1)U_{\text{norm}}$ and $E_2[U_1] = \alpha_2 U_1 + (1-\alpha_2)U_{\text{norm}}$. In words, the expectation of the partner's utility is a weighted sum of the preferences of the “average" person and the individual's actual preferences. This might be the case of two friends whose preferences are far from the norm, and therefore would not choose to use their own preferences in place of the other, as in Smithian Altruism. Assume that α_1 and α_2 are chosen by person 1 and person 2, but that there is a cost associated in doing so. In simple terms, each can learn about the other and better understand the reactions of the other person, but only by putting forth effort and time. It is unlikely that $\frac{\partial U_1}{\partial E_1[U_2]}$ is independent of α_2, but as a first approximation to the benefits of getting to know someone, it may be acceptable.

With this in mind, assume that the utility costs of maintaining α_i are a differentiable, increasing, and convex function $Z_i(\alpha_i)$. Therefore, with optimal behavior, we would have individual 1 choose α_1 such that, if we exclude corner solutions for the moment,

$$\frac{dZ_1}{d\alpha_1} = \frac{dU_1}{d\alpha_1}$$

Let us assume that neither person will be giving gifts as a result of getting to know a person better, i.e. $\frac{dc_i}{d\alpha_j} = 0$.[21] For simplicity, I also assume $\frac{dU_{\text{norm}}}{d\alpha_1}$ and $\frac{dU_{\text{norm}}}{d\alpha_2}$ are zero. With no gifts, this is equivalent to saying $\frac{\partial U_{\text{norm}}}{\partial E[U_i]}$ is zero. In words, the average person does not care *that* much about others.

I also assume $\frac{dU_1}{dc_1} > 0$, $\frac{\partial U_1}{\partial c_1} > 0$, and $\frac{\partial U_{\text{norm}}(c_1)}{\partial c_1} > 0$.

First Law of Attraction. *With the above assumptions, an altruistic individual will only learn about another person's preferences if this person has a utility higher than the normal person's would be. It follows that, without the ability to give gifts, an altruistic individual would prefer to remain ignorant about a sadder-than-average*[22] *person's preferences.*

[21] This assumption may be a bit hard to swallow. Essentially I am looking at who one might make friends with, even if they cannot trade gifts.

[22] Given his consumption.

Proof. Consider the following.

$$\frac{dU_1}{d\alpha_1} = \frac{\partial U_1}{\partial E_1[U_2]}\left(U_2 + \alpha_1 \frac{dU_2}{d\alpha_1} - U_{\text{norm}} + (1-\alpha_1)\frac{dU_{\text{norm}}}{d\alpha_1}\right)$$
$$\frac{dU_2}{d\alpha_1} = \frac{\partial U_2}{\partial E_2[U_1]}\left(\alpha_2 \frac{dU_1}{d\alpha_1} + (1-\alpha_2)\frac{dU_{\text{norm}}}{d\alpha_2}\right)$$
$$\Rightarrow \frac{dU_1}{d\alpha_1} = \frac{\partial U_1}{\partial E_1[U_2]}\left(U_2 - U_{\text{norm}} + \alpha_1\alpha_2 \frac{\partial U_2}{\partial E_2[U_1]}\frac{dU_1}{d\alpha_1}\right)$$
$$\Rightarrow \frac{dU_1}{d\alpha_1} = \frac{1}{1-\alpha_1\alpha_2 \frac{\partial U_1}{\partial E_1[U_2]}\frac{\partial U_2}{\partial E_2[U_1]}}(U_2 - U_{\text{norm}})$$

In the above, $\alpha_1\alpha_2$ is already ≤ 1. We want to show, however, that the entire denominator is greater than zero. Using the above assumption that $\frac{\partial U_{\text{norm}}}{\partial E[U_i]} = 0$,

$$\frac{dU_1}{dc_1} = \frac{1}{1-\alpha_1\alpha_2 \frac{\partial U_1}{\partial E_1[U_2]}\frac{\partial U_2}{\partial E_2[U_1]}}\left(\frac{\partial U_1}{\partial c_1} + \alpha_1(1-\alpha_2)\frac{\partial U_1}{\partial E_1[U_2]}\frac{\partial U_2}{\partial E_2[U_1]}\frac{\partial U_{\text{norm}}(c_1)}{\partial c_1}\right)$$

With the earlier assumptions, we see that in order for $\frac{dU_1}{dc_1} > 0$, $\frac{1}{1-\alpha_1\alpha_2\frac{\partial U_1}{\partial E_1[U_2]}}$ must be greater than 0. With this, we see that $\frac{dU_1}{d\alpha_1}$ increases as $U_2 - U_{\text{norm}}$ increases. Going back to the equilibrium condition where $\frac{dZ_1}{d\alpha_1} = \frac{dU_1}{d\alpha_1}$, we see that a larger $\frac{dU_1}{d\alpha_1}$ means a larger $\frac{dZ_1}{d\alpha_1}$. As Z_1 increasing and convex, we see that this implies Z_1 is larger, which implies α_1 is larger. Furthermore, if $U_2 - U_{\text{norm}} < 0$, then as Z_1 is increasing, this implies a corner solution; specifically $\alpha_1 = 0$. □

There were a number of potentially unreasonable assumptions made here, but first I want to provide a few examples of what this theorem predicts. Specifically, let us suppose that a rich man is presumed to be happy. If the assumptions are valid and the model correct, a sad rich man would have no friends, or at least no one would voluntarily learn his preferences. However, we made an unreasonable assumption that gifts cannot be given. If this were allowed, a sad rich man may still have "friends" that hope to receive gifts.

There might be some cyclical nature to this. A sad teenager might have trouble making friends, which might lower his utility even more. We have also been considering cases of altruism. It seems rather straightforward that if we considered cases of envy, we would have the sad individuals who are the desired "friends". It should be noted that this result does not mean that poor people cannot be befriended (i.e. a high α_1). It is relative to the person's consumption; if a normal person would be sadder with the same level of consumption, then one would want to get to know the poor, but happy, man.

First, I assumed perfect knowledge of consumption, which is unlikely. I also assumed that there cannot be any gifts. I expect that if this assumption was relaxed, a relatively poor individual might befriend (or try to befriend) relatively rich, but sad, individuals in order to procure gifts. I have also assumed that the "normal" person would not have any altruism and that the altruism, $\frac{\partial U_1}{\partial E_1[U_2]}$, is independent of the level of knowledge, α_1. I also assumed that goods cannot be shared or given, which may be an important motivator to friendship, and that the individual does not estimate using Smithian Altruism. Despite these onerous assumptions, I believe this to be an interesting line of inquiry with a multitude of extensions; hence this result is titled the *First* Law.

5 Concluding Remarks

As we have shown, it is difficult to justify both individuals strictly preferring to give divisible gifts unless we accept utility forms in which the other person's consumption enters directly. I believe the recursive forms of altruism are better descriptors of altruism and this might also explain why it is somewhat uncommon to see two individuals arguing over who will pay for dinner (beyond the limits of politeness).

Even with this result, economic altruism has many theoretical boundaries to overcome. Concepts like politeness, utility information transfers, risk-aversion sharing, spatial clustering of friends and family, and why some friendships flourish, while others fail, are all related to altruism but are incorporated at a rudimentary level. Furthermore, the theoretical results of imperfect altruism and imperfect knowledge of consumption have not been fully fleshed out. This paper has tried to address some of these concepts, but each could easily be a paper on it's own. As a result, I have tried to point out extensions along the way which I hope to complete in future works. Hopefully, future theoretical work can lead to previously unforeseen empirical tests for altruism as well.

That being said, this paper does introduce new theoretical constructs that combine altruism with imperfect information. Aside from the mathematical aspects of this paper, we also discussed possible explanations for why children might give gifts and how altruism may be an important factor in our concept of what actions are “normal" or “acceptable". One reason for learning more about others was provided; specifically, that altruistic individuals are interested in happier-than-average individuals. This seems to match up with contemporary advice for making friends (“Smile!"). We also briefly touched upon the possible effects of improved communications and implications for long-distance relationships. Hopefully these discussions have provided the reader with a greater admiration for the role of economic altruism in our daily lives and provoke more attention to the subject.

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